Preface

The main purpose of this book is to present a systematic and algorithmic perspective on the Monte Carlo method for solving partial differential equations (PDEs), especially their fast algorithms and practical applications. We will focus mainly on elliptic PDEs, i.e., the Laplace and Poisson equations describing equilibrium states. Moreover, the applications that motivate us will mainly come from microelectronic integrated circuit.

The Monte Carlo method in the Russian literature is often referred to (in translation) as "the method of statistical sampling." This phrase has much more meaning than "the Monte Carlo method," which was chosen due to the famous casino in Monte Carlo. Monte Carlo can be used to solve a variety of problems, and is the basis of the areas of stochastic ordinary differential equations (ODEs) and PDEs, and uncertainty quantification. In this book, we focus on the Monte Carlo method for solving partial differential and integral equations based on random walks. The random walk method essentially involves the Markov process.

Historically it is most interesting to realize that there are certain application areas where Monte Carlo has become the dominant numerical method. One such area is in financial computations. Another is a topic in this book, the title of which is *Monte Carlo Methods for Partial Differential Equations With Applications to Electronic Design Automation*. Electronic Design Automation (EDA) is the process of designing microelectronic circuits. It is a key technology in an enormously important component of modern information technology. When one has a putative circuit design, before it can be processed for fabrication various validation computations are done to make sure that the design is not flawed. One such computation is based on electrostatics and insures that electronic crosstalk or even more catastrophically, electric arching does not make the device malfunction. The various physical components of the circuit, especially interconnect wires, have their mutual (coupling) capacitances which must be computed. This process is known as capacitance extraction or capacitance calculation, and is a common computation in electrostatics. Since the equations of electrostatics are the Laplace or Poisson equations, methods for the solution of elliptic PDEs are employed. For many years, deterministic techniques were employed in capacitance extraction. However, it was noticed that computing capacitance via

Monte Carlo methods was a very rapid technique. Eventually, the commercial codes for capacitance extraction began replacing deterministic with Monte Carlo methods. This book is meant to give the reader the background and state-of-the-art computational knowledge to understand and use Monte Carlo methods for capacitance calculation problems in particular, and more generally for the solution of elliptic PDEs in EDA.

The work presented in this book are mostly from research projects undertaken by the Numbda group led by Wenjian Yu at the Department of Computer Science and Technology, Tsinghua University, China. All chapters except Chap. 3 are composed/edited by Wenjian Yu. Chapter 3 is from the collaborative work with Michael Mascagni of Florida State University and the National Institute of Standards and Technology (NIST). It is interesting to note that this chapter deals with a Monte Carlo methods for the telegrapher's equation, which is a hyperbolic PDE and definitely not describing an equilibrium problem. Most content of this book is based on the original publications shared at https://numbda.cs.tsinghua.edu.cn/paper.html.

We want to emphasize that the book is by no means intended to be comprehensive. The absence of coverage of related work should by no means diminish their value and contribution. Many academic or industrial experts have made significant contributions in the field, and the reader is encouraged to investigate their work. Publications that publish results in the area of Monte Carlo methods for PDEs include international journals on applied mathematics such as *Journal of Computational Physics*, *SIAM Journal on Scientific Computing*, and *Monte Carlo Methods and Applications*.

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