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Analytical-BEM coupling method for fast 3-D interconnect resistance extraction

Abstract Deep submicron process technology is widely being used and interconnect structures are becoming more and more complex. This means that the resistance calculation based on two-dimensional models can no longer provide sufficiently accurate results. This paper presents a three-dimensional resistance calculation method called the combined analytical formula and boundary element method (ABEM). The method cuts selected interconnecting lines then it calculates the resistances of straight sections using an analytical formula and the resistances of the other sections using the boundary element method (BEM). The resistances of the different sub-regions are combined to calculate the resistance of the entire region. Experiments on actual layouts show that compared with the commercial software Raphael based on finite difference method, the proposed method is 2–3 orders of magnitude faster. The ABEM method uses much less memory (about 0.1%–1%), and is more accurate than Raphael with default mesh partitions. The results illustrate that the proposed method is efficient and accurate.

Keywords Very large scale integration, Interconnecting resistance, 3-D extraction, Analytical formula, BEM

1 Introduction

Interconnect parasitic resistance is a factor determining circuit performances such as time delay. Since the 1970s, the extraction of interconnect resistance has been an important issue. At a current frequency of $4-5$ GHz, the direct current (DC) resistance model of interconnects is still

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valid. For example, DC resistance is sufficiently accurate in some sense, such as for time delay analysis [1].

There are usually six to eight levels of wires in VLSI circuits, and many vias are used to connect different levels. Interconnect structures are becoming more and more complex. Fig. 1 gives a structure from actual layouts [2]. For such complex structures, 2-D extraction or approximate formulas used for extraction are probably inaccurate. Moreover, in the present deep submicron process technologies, the bulk resistivity of narrow wires with different widths or lengths [3] is not constant, which makes it more difficult for the above extraction methods to handle actual structures. Therefore, 3-D calculation is necessary, and similar discussion can be found in Ref. [4].
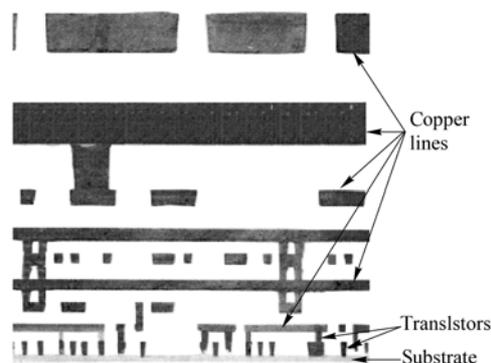
Fig. 1 An eight-level interconnect structure [2]. Many vias are used to connect different levels

There are roughly two kinds of methods to calculate resistance. One is the analytical formula $R = \rho L/S$, which is accurate only for two-terminal straight wires. The other is the numerical method, such as the finite difference method (FDM) [5], finite element method (FEM) [6], and BEM[7-8]. FDM and FEM discretize the entire volume of resistance regions, producing a great deal of variables. BEM, with fewer variables, higher accuracy and stronger ability to handle complex structures, has received more and more attention [7-8]. Note that the BEM in Refs. [7-8] is 2-D

In this paper, we present a 3-D extraction method for DC

resistance. The method is the combination of the analytical formula $R=\rho L/S$ with BEM. After a resistance region is partitioned into two kinds of sub-regions called uniform and non-uniform sub-regions, the resistance of these sub-regions is obtained using the formula and BEM, respectively. Then, the two kinds of resistance results are coupled into the overall resistance of the original region. Numerical experiments on realistic interconnects show that the method occupies fewer computational resources, but preserves high accuracy.

The rest of the paper is organized as follows. In Sect. 2, the principle of interconnect resistance calculation is outlined. In Sect. 3, the coupling method is presented in detail, including partitioning the resistance regions, calculating the resistances separately, and finally coupling the different kinds of resistance. In Sect.4, numerical experiments are presented. Finally, we give conclusions in Sect. 5.

2 Interconnect resistance calculation

2.1 Resistance of straight conductors

For a uniform structure of a straight conductor, it is quite straightforward to calculate its resistance between its two terminals C_1 and C_2, with the analytical formula:

$$R = \rho L / S \tag{1}$$

where ρ is the resistivity L is the length along the current flow direction, and S is the area of the cross section perpendicular to the current flow, as shown in Fig. 2. The formula is only accurate for such conductors. For general resistance regions with multiple terminals, the formula becomes invalid.
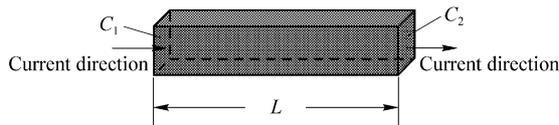
Fig. 2 A straight conductor with two parallel terminals C_1 and C_2. The resistivity is ρ. The Length along the current direction is L, and the cross section area is S. The resistance R can be obtained with the analytical formula

2.2 Equivalent resistance network of general regions

Generally, for a resistance region with p terminals ($p>1$), there is a resistor between each pair of terminals, and the equivalent resistance network is a full graph. Let the voltage of terminal j be 1 V (called main terminal), and voltages of the other terminals be 0 V. Then the resistor between terminals j and k, R_{jk} can be expressed as:

$$\frac{1}{R_{jk}} = \int_{\Gamma_{uk}} \sigma \frac{\partial u}{\partial n} d\Gamma = \int_{\Gamma_{uk}} \sigma q d\Gamma \tag{2}$$

where Γ_{uk} is the boundary of terminal k and σ is the conductivity (reciprocal of resistivity). u is the electric potential and q is the electric flux. The above calculation repeats for p times with different main terminals, and the complete resistance network of the region can be obtained.

Besides FDM and FEM, BEM can also solve numerically for the unknown q needed in Eq. (2). Due to fewer variables and smaller coefficient matrices, BEM is supposed to be very efficient.

2.3 Resistance calculation using BEM

To get the field q distribution in a homogeneous region, it is accurate to formulate the region in terms of the Laplace equation about the electric potential u

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \tag{3}$$

with mixed boundary conditions [7-8].

The application of the fundamental solution of the Laplace equation and Green's formula transforms Eq. (3) into a boundary integral equation [7]

$$c_s u_s + \int_{\Gamma} q^* u d\Gamma = \int_{\Gamma} u^* q d\Gamma \tag{4}$$

where u_s is the electric potential at the source point s, and c_s is a constant depending on the geometry of the boundary in the neighborhood of the point s. Γ is the entire boundary of the region. u^* is the fundamental solution of the Laplace equation. q^* is $\partial u^*/\partial n$, and n is the unit normal vector of Γ. Substitute boundary conditions, and we get a linear system:

$$Ax=f \tag{5}$$

where x includes discrete u and q variables on Γ. Solving the linear system, we can directly get q, and in turn the resistance from Eq. (2).

If the region is with several different resistivities, it can be cut into several regions, which are with only one resistivity respectively. In each of the regions, there is a Laplace equation (Eq. (3)). Then it can be inferred that BEM is able to treat complex structures both in geometry and resistivity. The test cases in Sect. 4 are with multilevel conductors and vias, which are with more than one values of resistivity.

In BEM, the number of variables usually increases with the size of calculated structures, and then building and solving the linear system Eq. (5) takes up much CPU time and computer memory. However, if the simulated structure is condensed, the BEM will be more efficient.

3 Analytical-BEM coupling method

In order to improve the BEM for large general structures, we reduce the size of the numerically calculated structures while preserving the accuracy. We can partition the entire resistance region into two kinds of sub-regions, with only one of which is to be numerically calculated using BEM. Since the question scale becomes much smaller, the BEM

calculation can be very quick. Such numerically calculated sub-regions are called non-uniform sub-regions. Examples include the corner sections of conductors. The other is called uniform sub-regions, which will be with uniform (or nearly uniform) current density, and its resistance can be obtained with Eq. (1). The conductor in Fig. 2 is such an example. The two sub-region resistance networks are coupled into the final one in the end. In this way, we couple the analytical formula with BEM.

3.1 Partitioning the original resistance regions

At low voltage frequency, the density of current flowing through corner sections of interconnects is not uniform. But it becomes nearly uniform some distance (equaling the width of the conductor at most) away from the corners [9]. According to this, we can partition the region into uniform and non-uniform sub-regions and distinguish them.

Since resistance regions are composed of conductors, we put emphasis on cutting conductors here. For simplicity but without losing universality, assume that current flows through a conductor horizontally, and the conductor is to be cut along this direction; The conductor width is w, and the cutting distance is d. By experiment, we find out that when $d=0.5w$ the coupling method is very efficient, and we employ this value in the following text.

Firstly, we identify conductor interfaces. Secondly, we extend the interface polygons horizontally in both directions by d. Thirdly, we combine all the polygons, and get some large enough rectangles. Lastly, we cut each of the rectangles as a whole from original conductors separately. Among the created sub-conductors, those containing rectangles are non-uniform sub-regions, and the sub-conductor between them constitutes a uniform sub-region on its own.

Here is an example. In Fig. 3, region conductor P_1 is to be cut. P_2, P_3, P_4 and P_8 are in the same plane as P_1; P_5 is above P_1, and P_6, P_7 is below it. $L_1 < 2d$, $L_2 > 2d$. The region is finally partitioned into three non-uniform sub-regions (N_1, N_2 and N_3) and two uniform ones (U_1 and U_2). The sub-regions of an actual interconnect structure are shown in Fig. 4.

3.2 Calculate resistance networks of the sub-regions

The resistance (networks) of uniform sub-regions can be obtained with the analytical formula (Eq. (1)).

The resistance networks of non-uniform sub-regions can be obtained using BEM.

In BEM application, we can improve its efficiency while not sacrificing accuracy. The improvement can be accomplished by the following techniques.

As we know, if a conductor within the calculated region is not in the current flow path, it is equipotential everywhere.

Such a conductor is of no use for numerical calculation for resistance. Removing it from the region affects neither the current value nor the resistance value, but is able to reduce the element and variable number. Such a "waste" conductor is marked in Fig. 4. In realistic BEM calculation, such waste conductors are removed, i.e., are not considered during the calculation.
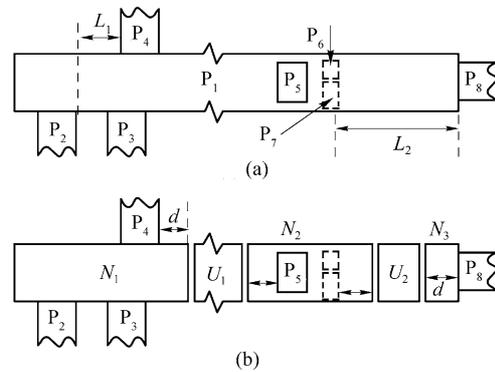
Fig. 3 Partition a conductor P_1 (top view). (a) Orginal; (b) After atting
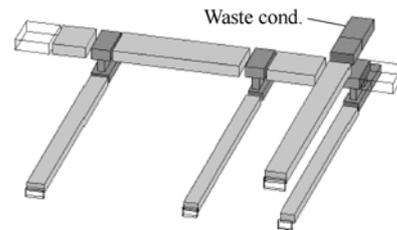
Fig. 4 Sub-regions of a realistic interconnect structure. Non-uniform sub-regions are in black grey. The virtual terminals are connectors between different sub-regions

Generally speaking, we only need to partition elements in the direction where potential u and flux q are varying. At the same time, the current flows perpendicularly through an interface between different conductors. If a surface is in turn perpendicular to the interface, we only partition the elements in one direction. As an example, see the conductor B in Fig. 5. The current is expected to flow horizontally along the bottom surface. Then the elements partitioned are only arranged in one direction, as shown with the dashed lines in the figure.
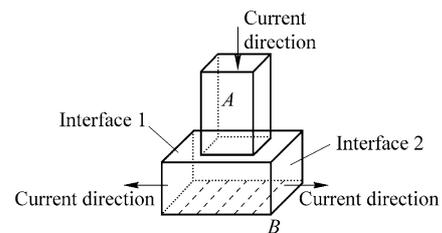
Fig. 5 Divide elements only in one direction

3.3 Couple sub-region resistance networks into the final network

The resistance networks of uniform and non-uniform regions can constitute a temporary net $\overline{G}$, which consists of some internal vertexes with unknown voltages (see Fig. 6). These vertexes are called virtual terminals, and the other terminals are original terminals. If the virtual terminals are eliminated, the final resistance network can be obtained.

Here we give a brief algorithm to get the final resistance network through eliminating the virtual terminals.

1) Let one original terminal be the main terminal (with voltage of 1 V), and the others be subordinate ones (with voltage of 0 V).

2) Assume non-uniform sub-region i (i=1, 2, N_n) has n_i terminals (including $\overline{n}_i$ virtual ones), and its resistance network G_i has $\overline{n}_i$ virtual vertexes that correspond to the virtual terminals. Give a voltage variable to each virtual vertex.

3) In $\overline{G}$, the current flowing through an edge can be expressed in terms of known voltage values and/or unknown voltage variables, and there is a current equation for each virtual vertex, i.e., Kirchhoff's current law (KCL) equation.

4) Solve the above $\sum\limits_{i=1}^{N_n} \overline{n}_i$ equations, and we get all voltage values and current values.

5) The current flowing through the original terminals can be obtained easily, and its reciprocal is the resistance.

6) Repeat with different main terminals, and we can get the resistance network of the original region.
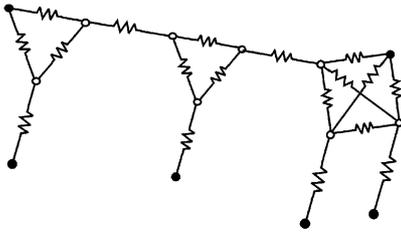
Fig. 6 The net $\overline{G}$ constituted by resistance networks of the sub-regions in Fig. 5, where the internal vertexes are in light grey. The final resistance network will be a complete graph with six vertexes

3.4 Overall flowchart of the coupling method

1) Partition the original resistance region, to create some uniform and non-uniform sub-regions.

2) Calculate the resistance of uniform regions with the analytical formula.

3) Get the resistance of the non-uniform regions with the improved BEM.

4) Combine the above two kinds of resistances with the algorithm in the forgoing subsection.

4 Numerical experiments

In order to examine the efficiency of the combined analytical–BEM method presented here (abbr. ABEM), we compare it with the pure direct boundary element method without using the analytical formula (abbr. DBEM), and Raphael RC3 [10] (version 2000.2). Raphael employs a finite difference solver with advanced non-uniform meshing scheme. We take two test cases from actual layouts, with each having one and two levels of vias. The region conductors in the cases are with different resistivities. Their minimum wire width is 0.09 μm.

For the two cases, the results of different programs are shown in Tables 1 and 2 respectively. All programs run on Sun Ultra Enterprise 450 (CPU frequency 248 MHz). Note that Raphael uses its default mesh grids. The results of Raphael with dense meshes of 2×10^6 grids are regarded as the criteria to obtain the "Error" of the ABEM, DBEM and Raphael. The maximum relative error of the diagonal entries in the resistance matrix can also be found in Tables 1 and 2.

Table 1 Results of case 1

	GridNum	Mem/M	Time/s	Error/%	Speed-up
ABEM	186	0.10	0.51	0.78	199.5
DBEM	870	4.71	4.68	6.75	21.7
Raphael	63 168	21.00	101.73	1. 05	1

Table 2 Results of case 2

	GridNum	Mem/M	Time/s	Error/%	Speed-up
ABEM	607	0.17	2.38	−1.17	2 570.9
DBEM	3 855	8.22	128.53	7.31	47.6
Raphael	336 398	89.00	6 118.76	1.58	1

From Tables 1 and 2, we can find that ABEM is tens of times faster than DBEM. When compared with Raphael, ABEM has a speedup of hundreds of to thousands of, while preserving accuracy. The memory usage of DBEM or Raphael is tens of times more. As for running accuracy, the ABEM performance is much better than Raphael with default meshes. All these reveal that the presented coupling method is able to produce accurate results and consumes much less computer time and memory.

5 Conclusions

In this paper, we present a 3-D interconnect resistance extraction method, which combines the analytical formula with the BEM. The original resistance region is partitioned into uniform sub-regions and non-uniform ones. Their resistance networks are obtained using the analytical formula and BEM respectively. Then, all the sub-region networks are transformed equivalently into the final network of the entire region. The numerically simulated domain is greatly reduced. Furthermore, computational time and memory usage are considerably condensed. The

accuracy is very high, and one reason is the reasonable application of the analytical formula.

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