

A Hybrid Random Walk Algorithm for 3-D Thermal Analysis of Integrated Circuits

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Abstract – In this work, a hybrid random walk method is proposed for the thermal analysis of integrated circuits. Preserving the advantage of generic random walk method (GRW), i.e. the suitability for simulating local hot-spots, the proposed techniques largely reduce its runtime for accurate high-resolution simulation, and is suitable for the realistic pyramid-shape IC model. This is achieved by combining the GRW and the floating random walk techniques, and a novel usage of rectangular cuboid transition domain. The techniques to handle the Neumann boundary and convective boundary in thermal simulation are also discussed. Numerical experiments on several IC test cases validate the efficiency and accuracy of the proposed techniques, and demonstrate more than 100X speedup over the GRW method.

I. Introduction

The continuous scaling trend of the CMOS technology has led to the drastic increase of the number of devices in integrated circuit (IC) and the ratio of interconnect delay to device delay. The heat dissipation has become a problem that threatens circuit reliability and performance [1]. Specifically, accurate and efficient chip-level thermal analysis is also indispensable for many design-time circuit optimizations.

Some thermal analysis algorithms have been proposed for chip-level analysis, such as the geometric multigrid solver [2], Green's function based fast algorithm [3], and the preconditioned conjugate gradient (PCG) algorithm [4]. The volume discretization has been employed to transform the problem into a linear equation system, and the temperature profile of the whole simulated domain is solved in most existing works. However, the entire temperature profile is not really required, because what we want is often the temperature of hot-spots at IC device layer.

Different from the deterministic methods solving linear equations, another kind of method for thermal analysis is the random walk method. Generally speaking, the random walk method is most efficient when point values or linear functionals of the solution are needed. Therefore, it should be suitable for the thermal analysis where the temperatures of some target hot-spots are needed. The random walk algorithms have been proposed for power grid analysis [5-7]. Based on the similarity of thermal analysis and power grid

analysis, the random walk techniques in [5] was applied to the problems of thermal via planning [8] and thermal analysis [17]. In [6], the importance sampling technique was proposed to accelerate the convergence rate of the random walk based power grid analysis. Besides, the random walk algorithms have also been successfully applied to capacitance extraction [9, 10, 18].

A major limitation of many existing works on thermal analysis [2, 3, 8] is that they consider a rectangular simulation domain, with a simplified boundary assumption (often the Dirichlet condition) accounting for the effect of heat dissipation. In practice, the heat spreader and heat sink attached to the die are much wider than IC die (see Fig. 1). By approximating the whole thermal system with a single rectangular domain, substantial error (up to tens of degrees in temperature) may be introduced [4]. In a recent work [4], a preconditioned conjugate gradient (PCG) algorithm was proposed for the realistic pyramid IC model, which takes the solution of a larger and approximate rectangular domain by fast Poisson solver as the preconditioner.

In this work, we propose a fast random walk algorithm for thermal analysis. The pyramid-shape IC model is considered. We focus on the application where only the temperatures of target hot-spots are needed, such as that in the floorplanning stage. The proposed method takes advantages of generic random walk (GRW) and floating random walk (FRW), to reduce the length of each walk. Employing pre-characterization techniques to handle the Neumann boundary and convective boundary, we further accelerate the hybrid random walk method. Compared with the generic random walk algorithm [5, 8], the proposed method achieves several orders of magnitude speedup while preserving high accuracy.

II. Background

A. 3-D Thermal Model

In the pyramid-shape IC model (see Fig. 1), the IC region mainly includes two parts: silicon substrate and the interconnect layer. The former is made of silicon, while the latter is filled with metal and dielectrics. To simplify the

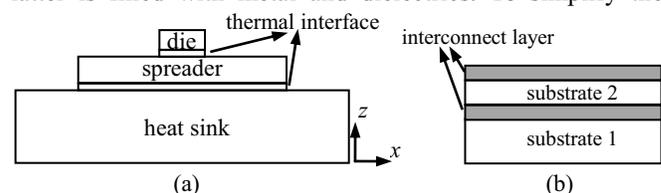


Fig. 1. The pyramid-shape IC geometry for thermal simulation: (a) the side view, (b) the details of a IC with two tiers of dies.

This work was supported in part by NSFC under Grant 61076034, Tsinghua University Initiative Scientific Research Program, and Fundamental Research Funds for the Central Universities under Grant 2011JBZ002.

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discussion, we consider the interconnect layer as a homogeneous layer with an effective thermal conductivity calculated using the percentage of metal volume. This is typical in most previous works. Note that there is no problem for the random walk method to handle the model with lateral variation of thermal conductivity as considered in [8]. Similarly, we ignore the heat dissipation through packaging and board in this work, since that is a minor dissipation channel.

The steady-state thermal analysis involves solving the temperature profile $T(x,y,z)$ from the 3-D Poisson equation:

$$k \left[\frac{\partial^2 T(x,y,z)}{\partial x^2} + \frac{\partial^2 T(x,y,z)}{\partial y^2} + \frac{\partial^2 T(x,y,z)}{\partial z^2} \right] = -p(x,y,z), \quad (1)$$

where k is thermal conductivity and $p(x,y,z)$ is the internal heat generation density at point (x,y,z) . The heat generation is due to the device modules, or function blocks located around the top surface of the silicon die. Eq. (1) holds for a homogeneous region. For a problem with multiple homogeneous regions, the equation of continuous heat flux should be applied at the interface between two regions.

The finite volume method (FVM) is conventionally used for 3-D thermal simulation, where the domain is discretized into cells and each cell is associated with a temperature [1-4]. Similar to simulating the steady-state electric current field with electric resistors, we can define and calculate thermal resistor to model the heat flow through the interface between any two adjacent cells. Fig. 2 illustrates the finite volume discretization of a 3-D rectangular domain. For two cells with different thermal conductivities or different length along the aligned direction, the thermal resistance connecting them can be calculated as the series of resistors. For example, at the interface of two homogeneous layers [see Fig. 2(c)], the across-interface thermal resistance R_{tz} is:

$$R_{tz} = \frac{1}{2k_1} \cdot \frac{h_{z1}}{h_x h_y} + \frac{1}{2k_2} \cdot \frac{h_{z2}}{h_x h_y}, \quad (2)$$

where h_x and h_y are edge sizes of cell along x -axis and y -axis, respectively. h_{z1} and h_{z2} are the heights of the two adjacent cells; k_1 and k_2 are the thermal conductivities.

The heat source resembles the current source in electric circuit. Thus, an equivalent circuit with resistors and current source is generated. With the nodal analysis approach, a linear equation system:

$$\mathbf{AT} = \mathbf{f} \quad (3)$$

is formed, where \mathbf{A} is a sparse symmetric positive definite

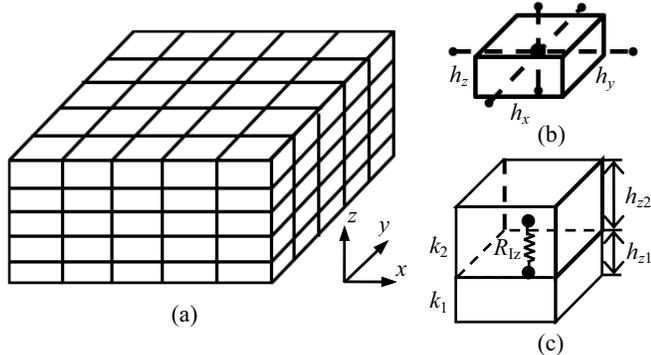


Fig. 2. A rectangular domain is discretized into grid cells, and each cell is assumed with constant temperature. (a) The discretization of a homogeneous domain. (b) A cell and six thermal resistors connecting it to its neighboring cells. (c) Two cells adjacent to the interface of two subdomains with different thermal conductivities.

matrix, \mathbf{f} is the vector of current sources, and \mathbf{T} is the temperature vector. The temperature profile can be obtained by solving (3) with linear equation solvers.

There are different boundary conditions for the simulation domain. At the bottom surface of heat sink, a convective condition should be set, which models the heat transfer mechanism at the interface of heat sink and air:

$$k \frac{\partial T}{\partial \bar{n}} + h(T - T_{amb}) = 0, \quad (4)$$

where \bar{n} is the out normal direction of the boundary, T_{amb} is the ambient temperature, and h is the convective coefficient. The partial derivative in (4) can be approximated with finite difference formula. By defining

$$R_{amb} = 1/(h \cdot h_x \cdot h_y), \quad (5)$$

where h_x and h_y are the edge sizes of cell along x -axis and y -axis respectively, we can model the effect of convective boundary with the thermal resistors of value R_{amb} . They connect the nodes of boundary cells to a virtual node with temperature T_{amb} . For other boundaries of the domain, Neumann boundary (adiabatic condition) is usually assumed. It is naturally modeled by the equivalent circuit.

B. The Random Walk Method

There are mainly two kinds of random walk method: the discrete random walk (DRW) method (also called “fixed” random walk method) [5-8], and the floating random walk (FRW) method (also called “walk inside the domain” method) [9-11]. The DRW method relies on an existing mesh grid, where walk jumps from a grid node to its adjacent nodes. In the FRW method, the cubic [9, 10] or spherical [11] transition domains with variable size are employed, and each step of walk is from the center to the boundary of a transition domain. As shown in Fig. 3, a walk in DRW method usually involves larger number of steps than a walk in FRW method.

The DRW has been applied to the problems of power grid analysis and thermal analysis [5-8], which is called *generic random walk* (GRW) method. In a game of GRW, a walker starts at a node in the grid, for which the voltage/temperature is calculated. The walker then randomly visits a neighbor node. The probability of each neighbor j being chosen from node i is

$$P(i,j) = \frac{G_{ij}}{\sum_j^{d(i)} G_{ij}}, \quad (6)$$

where $d(i)$ is the edge degree of node i , and G_{ij} is the electric/thermal conductance between node i and its neighbor j . At each node, the walker will receive a reward of

$$T_r(j) = \frac{p_j}{\sum_j^{d(i)} G_{ij}}, \quad (7)$$

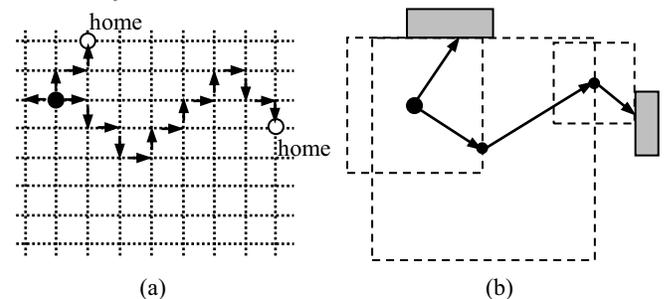


Fig. 3. Discrete random walks on a mesh grid (a), and floating random walks using cubic transition domains (b).

where p_j is the current/power injected into current node j . The walk ends if the walker hits a node with known voltage/temperature (home node), where the walker receives the final reward of the value of voltage or temperature. The total amount of money collected by the walker is an estimation of the voltage/temperature of the starting node. Due to the central limit theorem, the average money of many walks obeys a normal probability distribution, whose variance is inversely proportional to the number of walks. This gives a tradeoff between runtime and accuracy.

Several techniques have been employed to accelerate the GRW method. One is to set the limit on path length of a random walk. If suitably set, the time of performing random walk can be reduced with negligible error [5, 8]. Another technique is setting the cell whose temperature has been calculated as a new home cell. This cuts down the average length of walks significantly, and is very useful for solving the voltage/temperature for many nodes.

Note that, R_{amb} is larger than the thermal resistance of cells for several orders of magnitude, in the thermal model with convective boundary. Therefore, the random walk needs more steps to terminate, if compared with the thermal model with the Dirichlet boundary. This brings difficulty to the computational efficiency.

To improve the efficiency of GRW, a hierarchical random walk method was proposed in [5], whose basic idea is to divide the grid into a global grid and multiple local grids, and build transition matrix for each local grid. The transition matrix contains the transition probabilities among the external nodes of a local grid. Special technique is also proposed to sparsify the transition matrix [5], for reducing the time of performing random walk.

The FRW method is suitable for the Laplace equation or Poisson equation (1) without preset mesh grid in the simulation domain. For capacitance extraction, where the Laplace equation is solved, the FRW method has large advantages over the deterministic methods for problems involving a large number of conductors [9]. To fit the Manhattan interconnect geometries, cubic transition domain is used [see Fig. 3(b)]. And, to make the walks across dielectric interface easily, finite difference techniques have been presented in [9] to pre-characterize the transition cube with two dielectric layers. For the thermal problem governed by Poisson equation (1), the power item $p(x, y, z)$ brings difficulty to FRW. Only for some special cases, such as in [11] where the power density within the whole domain is uniformed, the FRW method can be efficiently used.

In the FRW method, the probabilities from the center point to the boundary panels of transition cube are stored. This costs much less memory than the hierarchical random walk method in [5], for which the transition probabilities among any two boundary nodes are needed. Because the transition domain in FRW method is usually scalable, the characterization of it can usually be performed offline. This means shorter runtime of FRW method than the hierarchical method. Because most portion of simulation domain of a thermal analysis problem is governed by the Laplace equation (i.e. no power item), the FRW method would be applied there to enable high performance.

III. A Hybrid Random Walk Algorithm for the Thermal Analysis

In this section, a hybrid random walk method for thermal analysis is proposed. It combines the generic random walk method and the idea of floating random walk. Techniques handling Neumann boundary and convective boundary are also discussed for the reduction of runtime.

A. The Basic Idea

The advantage of FRW over GRW is due to the hop from the center of a transition domain to its boundary, which would reduce the length of walk (see Fig. 3). Our idea to improve the GRW for thermal analysis is to perform FRW in the region without power dissipation. This is illustrated in Fig. 4, where we show a walk starting from the GRW region and then behaving in the FRW manner. Therefore, we get a hybrid random walk scheme, and avoid the difficulty of FRW for handling the power item in general heat problem.

The choice of transition domain affects the efficiency of FRW. In the problem of thermal analysis, the random walk is performed in the pyramid-shape IC geometry, where the silicon die has much larger width than its height. The transition domain of cuboid shape, rather than the cubic domain for capacitance extraction [9], should be used to achieve better efficiency. This enables jumping far in lateral directions, and reduces the length of walk. And, to make the walk crossing the interface of different thermal materials, we also need the cuboid transition domain with two halves of different materials. These two kinds of transition domains are labeled with "I" and "II" in Fig. 4. Note that the FRW transition domain is bounded by the boundaries of the geometric model, and should not intersect any power dissipation region. If the walk location is approaching to these boundaries, the FRW will be degraded to the GRW.

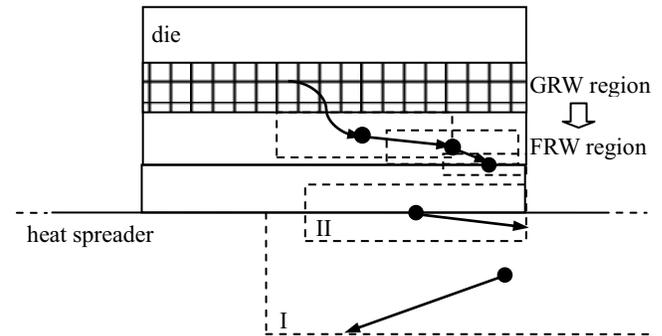


Fig. 4. A portion of the pyramid-shape IC model for the illustration of the hybrid random walk method. Several transition domains with cuboid shape are shown.

The aspect ratio of the cuboid transition domain may affect the performance of the hybrid random walk algorithm. We simply set the lateral size of the domain (length and width) to be $10\times$ of its height. In Section IV, we will present the experimental results on test cases with different sizes, which reveal how the aspect ratio of the cuboid affects the performance.

B. Scalable Transition Domain and Its Characterization

For an arbitrary transition domain where the Laplace equation of temperature $T(\mathbf{r})$ holds, we have:

$$T_c = \oint_S G_s(\mathbf{r}) T(\mathbf{r}) d\mathbf{r} \quad (8)$$

Here, T_c is the temperature at the domain's center point, and S is the boundary of the domain. $G_s(\mathbf{r})$ is the surface Green's function, which has non-negative value and can be regarded as the probability density function (PDF) for selecting a random point on S . If we discretize S into small panels, with $G_s(\mathbf{r})$ we can calculate the transition probabilities to these panels, which makes the FRW procedure feasible [9]. More importantly, these transition probabilities can be pre-calculated and recalled during the FRW procedure. This largely reduces the runtime of the FRW algorithm.

The surface Green's function $G_s(\mathbf{r})$ has the analytical expression only for the cubic or sphere transition domain with homogeneous material [9]. For the transition domain with other geometry or including inhomogeneous materials, we have to use numerical techniques to characterize its transition probabilities. Rather than using the finite difference techniques as in [9], we use the GRW method in this work to accomplish the characterization of transition domain. We make a dense discretization mesh on a cuboid transition domain, which corresponds to a resistance network. Then, random walks are started from the geometric center of this domain, and terminate at the domain's boundary. In the characterization, we perform M random walks, and store the numbers of their destination boundary cells in an array of length M . Because M can be view as the resolution of the probability estimation, we set it to be a large number in our experiment. While performing the FRW, for each hop we randomly pick an integer k from $[0, M)$. Then, the k -th element of the array indicates this hop's target. This approach enables much faster transitions in the FRW procedure than the conventional approach using the transition probabilities directly. The tradeoff may be the memory usage of the array for destinations, which is affordable in our problem of thermal analysis.

It should be pointed out that the transition probabilities are only relevant to the shape of the transition domain. In other words, the transition probabilities do not change if the domain is proportionally scaled. So, for an aspect ratio of the cuboid, we only need to pre-characterize a single transition domain. When performing a hop in FRW, we should firstly traverse all boundaries and power regions, and then choose the largest "safe" transition domain. By scaling the pre-characterized transition domain to this largest one, we can therefore execute the hop.

Different from the scalable FRW transition domain, another approach may be employing multiple fixed-size transition domains [5, 12]. This kind of transition domain is more generalized, and useful for handling complex medium and conductor topology. For the thermal analysis, the scalable transition domain is more efficient and costs much less memory storage than the approach using multiple fixed-size transition domains.

C. Handling the Neumann and Convective Boundaries

In the thermal model, Neumann boundary is assumed at the sidewalls. If a walk stops at the Neumann boundary, no FRW transition domain is available to continue the walk. In this case, it has to behave in the GRW manner, which definitely increases the length of walk. A remedy to this

situation is the path reflection technique [13], which can be illustrated as Fig. 5(a). Neumann boundary condition is equivalent to the situation where the original domain is mirrored with respect to the boundary. In such a way, the Neumann boundary is no more considered as an obstacle to the random walk. Note that it is not necessary to enlarge the original simulated domain in the mirroring operation. In fact, if the walk overcomes the boundary jumping to an external point, its position is transformed to the corresponding mirrored point inside the domain [see Fig. 5(a)].

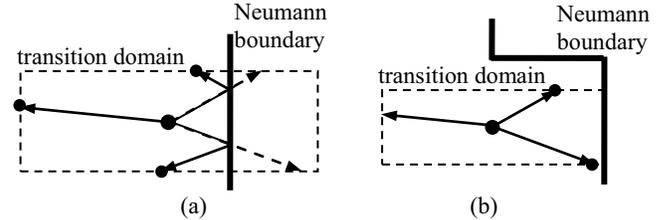


Fig. 5. Handle the Neumann boundary with the path reflection technique (a), or a special transition domain (b).

For some situations, the path reflection technique does not work. An example is shown in Fig. 5(b), where the inflated transition domain will intersect three Neumann boundary faces. In such cases, the equivalent mirrored domain for the original domain is not available. To settle this problem, we employ a special transition domain, which is restricted by Neumann boundary but avoid the walk stopping on it [see Fig. 5(b)]. This transition domain is defined as the one with a sidewall face being Neumann boundary. Using the similar GRW procedure described in last subsection, we can characterize the transition probabilities from the domain's center to the left five boundary faces. Combining the path reflection technique and the Neumann-specific transition domain, the efficiency of FRW procedure is preserved.

Another special boundary in thermal analysis is the convective boundary. If a walk stops at the convective boundary, a following GRW hop can terminate the walk in certain probability. However, this probability is very small due to the large value of R_{amb} . To overcome this difficulty, we can define a convective-specific transition domain. It has the same size as the normal transition domain touching the convective boundary, but forbids the walk stopping at the convective boundary. The walk jumps to the left five boundary faces, or terminates at the virtual node with temperature T_{amb} . The transition probabilities are pre-characterized also with the GRW procedure. It is obvious that this convective-specific transition domain helps to reduce the length of walk. However, from (5) we see that R_{amb} depends on the edge sizes of discretization cell, which means the variable convective-specific transition domain cannot fit this discretization sizes. This may introduce error, and we will evaluate it through the simulation experiments.

IV. Numerical Results

To validate the efficiency of the proposed random walk techniques, several chip structures with different power profiles are tested. The random walk algorithms have been implemented in C++. While characterizing each transition domain, $M=10^6$ random walks are performed. The efficient Mersenne twister random number generator [16] is used. Unless otherwise stated, we use the cuboid transition domain with aspect ratio of 10. For the basic hybrid random walk

implementation, two transition domains (described in Section III.A and III.B) are pre-characterized. For the treatment of Neumann boundary and convective boundary, two additional transition domains are needed.

The termination criterion of random walk algorithms is always that the $1-\sigma$ error becomes smaller than 1% of the mean value, equivalent to a relative error within 3% (in 99.7% confidence level). All experiments are carried out with serial computing on a PC with 2.70GHz dual-core Pentium CPU, 6GB memory.

A. Test Cases

The following chip structures are used in experiments.

- **Structure 1:** A four-core 2-D chip artificially generated, which is the Testcase no. 2 in [4]. The transverse dimensions of the die, spreader, and sink are $1\text{cm}\times 1\text{cm}$, $3\text{cm}\times 3\text{cm}$ and $7\text{cm}\times 7\text{cm}$. The thickness and material parameters are listed in Table I. The power map (totally 176W) models a scenario with one core idle, one core with peak load and two others with median loads (see Fig. 6).
- **Structure 2:** A 2-D chip imitating the POWER6 microprocessor [14]. Except that the die is of $1.6\text{cm}\times 2\text{cm}$, the transverse dimensions are the same as Structure 1. The thickness and material parameters are listed in Table I. The power map is similar to Testcase no. 1 in [4] (shown in Fig. 6), with total power of 175W.

The heat convective coefficient at the bottom surface of heat sink is $8700\text{ W}/(\text{K}\cdot\text{m}^2)$ [3]. For each case, the power sources are assumed to present in a $5\mu\text{m}$ -thickness layer at the top of substrate. The ambient temperature is set to 20°C .

TABLE I
The Thickness and Material Parameters

Thickness of	Structure 1 (μm)	Structure 2 (μm)	Thermal conductivity ($\text{W}/(\text{K}\cdot\text{cm})$)
Interconnect layer	50	10	0.3
Silicon substrate	450	480	1.25
Thermal interface	200	200	0.3
Heat spreader	1500	1500	3.95
Heat sink	5000	5000	3.95

For each test structure, with different resolution of discretization we obtained three test cases. Case 1-1, Case 1-2, Case 1-3 are obtained from Structure 1, while Case 2-1, Case 2-2 and Case 2-3 are from Structure 2.

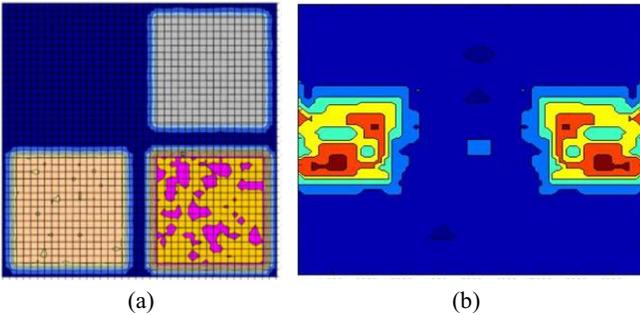


Fig. 6. The power maps used for (a) Structure 1 and (b) Structure 2.

B. Efficiency and Accuracy Validation

To evaluate the efficiency of proposed techniques, three versions of random walk methods are defined.

- GRW: The GRW method for thermal analysis [8].
- Hybrid0: The hybrid method combining GRW and FRW,

as described in Section III.A and III.B.

- Hybrid1: The hybrid random walk method using the techniques handling Nuemann boundary.
- Hybrid2: The hybrid random walk method using the techniques for handling both Nuemann and convective boundaries.

They are used to calculate the node temperatures on the power-source layer for each test case. The average runtime for a node is listed in Table II. In the second column of the table, we list the number of discretization nodes in each case. The average number of walks (#walk) performed for simulating a node and average number of hop in a walk (#hop) are also given.

TABLE II
The Average Runtime of Random Walk Methods for Calculating the Temperature of a Node (time in unit of second)

Test case	#node	GRW			Hybrid random walk				
		time	#walk	#hop	hybrid0	hybrid1	hybrid2	#hop	Sp [*]
1-1	5.24e5	49.8	5471	2.34e5	28.65	23.4	2.61	7.52e3	19
1-2	4.19e6	199	5522	9.28e5	48.2	33.68	3.66	8.11e3	54
1-3	6.55e7	949	6409	5.64e6	57.37	41.33	4.32	1.21e4	220
2-1	5.33e5	35.5	3576	2.52e5	32.86	29.46	1.46	1.08e4	24
2-2	4.26e6	143	3709	9.90e5	43.71	38.38	2.73	1.57e4	52
2-3	6.66e7	762	3281	5.98e6	72.7	48.41	5.04	2.71e4	151

*Speedup ratio of the hybrid2 to GRW.

From Table II, we can see that for the cases with coarse grid discretization there is no remarkable advantage brought by the hybrid random walk over GRW. This is because a hop in GRW is executed much faster than a hop in FRW. With the increase of discretization resolution, the runtime of GRW increases rapidly, since the length of walk in GRW depends on the number of grid nodes. Because the runtime of the FRW and proposed hybrid method hardly depends on the discretization of the domain, the advantage of proposed method over GRW becomes remarkable. For the largest cases (Case 1-3 and 2-3), the speedup ratios of Hybrid1 and Hybrid2 reach **23** and **220** respectively, while comparing with GRW. Actually, the number of hops per walk in Hybrid0 can be one order of magnitude fewer than that in GRW. After using the techniques handling Neumann and convective boundaries, the number of hops per walk is reduced further. The experiments also reveal that the hybrid random walk method hardly affects the number of walks for convergence.

While running Hybrid2, the pre-calculated transition probabilities should be loaded, which costs about 81MB in our experiments. The pre-characterization is executed with the approach given in Section III.B. In our implementation, we partition each side of the cuboid transition domain to 20 segments. Under this setting, the pre-characterization costs about 33 minutes which is affordable.

To validate the accuracy of the random walk methods, we use the Matlab “\” operator [15] to solve Case 1-1 and Case 2-1. The results are regarded as the standard value. For the other cases, the direct equation solver in Matlab “\” cannot complete the computation due to memory limitation. For Case 1-1, Matlab “\” costs 2198 seconds and 5.5GB memory, and the results show the highest temperature is 83.0°C . The highest temperature obtained from Hybrid1 is 86.0°C , whose error is consistent to the set accuracy criterion. For Case 2-1, Matlab “\” produces the result in 1579 seconds, with the

highest temperature of 61.0°C. For this cases, the highest temperature obtained from Hybrid1 is 63.1°C. For the temperatures of nodes in power region, the errors of Hybrid1 are also calculated. The results show that the average error is less than 0.7°C. The comparisons validate the accuracy of the hybrid random walk method.

The accuracy of Hybrid2 is also evaluated, which shows that its error can be as much as 20°C. After careful analysis, we find out that this is due to the mismatch of R_{amb} used in GRW (i.e. the FVM discretization) and R_{amb} essentially used with the convective-specific transition domains. Therefore, the treatment of convective boundary in Hybrid2 is not valid due to loss of accuracy.

Finally, to evaluate how the aspect ratio of the cuboid transition domain in FRW affects the efficiency of the proposed hybrid method, we change it from 10 and rerun the experiments for Case 1-3. With different kind of cuboid transition domains, the average runtime of Hybrid1 for a node is listed in Table III. From this Table, we can see that using the cuboid with aspect ratio of 10 brings **4.1X** speedup as compared with the cubic transition domain. And for the cuboids with similar aspect ratio, the performance of the hybrid random walk method is similar.

TABLE III

The Runtime of the Hybrid Random Walk Method with Different Aspect Ratios of Transition Cuboids (in unit of second)

Aspect ratio	1	5	8	10	12	15	20
Time/node	170	43.3	41.9	41.3	40.9	41.8	42.5

V. Conclusions

In this paper, a hybrid random walk method is presented for the thermal simulation with chip, heat spreader and heat sink regions. By combining the generic random walk on discretization grids and the floating random walk in medium domain, the hybrid method achieves large efficiency improvement for the thermal analysis of hot-spots in IC. Efficient techniques are proposed to pre-characterize and utilize a rectangular cuboid transition domain, and handle the Neumann boundary in the floating random walk procedure. Experimental results validate the proposed techniques and demonstrate its speedup of up to 23X over the generic random walk method, while keeping high accuracy.

In the future work, we will further investigate efficient and accurate technique to handle the convective boundary in thermal analysis. The fast random walk based thermal solver may also be applied to the floorplanning stage of IC design.

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