Parallel Statistical Capacitance Extraction of On-Chip Interconnects with an Improved Geometric Variation Model

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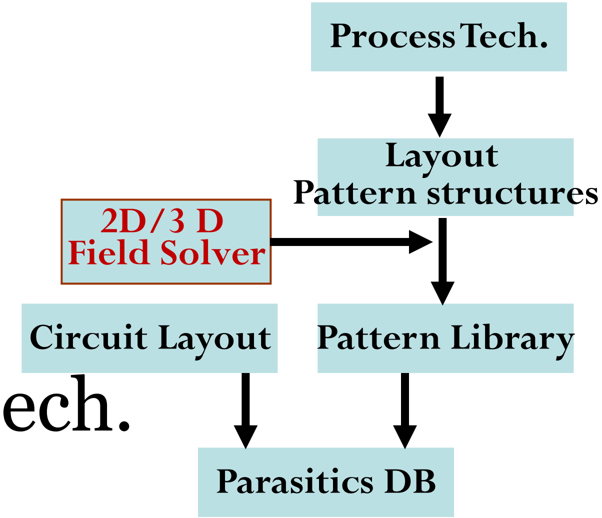
Outline

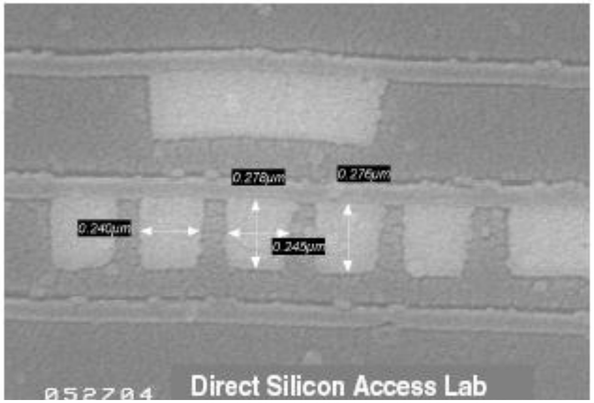
- Background
- Geometric Variation Models
- Experiments with Different Geometric Models
- A Parallel Statistical Capacitance Solver and Numerical Results
- Conclusions

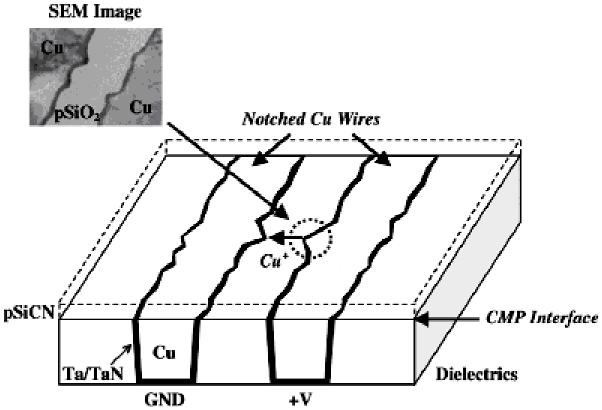
Background

- Parasitic (R, C) extraction
  - Crucial for interconnect modeling and accurate circuit analysis
  - In capacitance extraction, the field solver algorithms are important
- Process variations in nano-scale tech.
  - Geometric variations
  - Surface roughness, spatial correlation

Flowchart of LPE

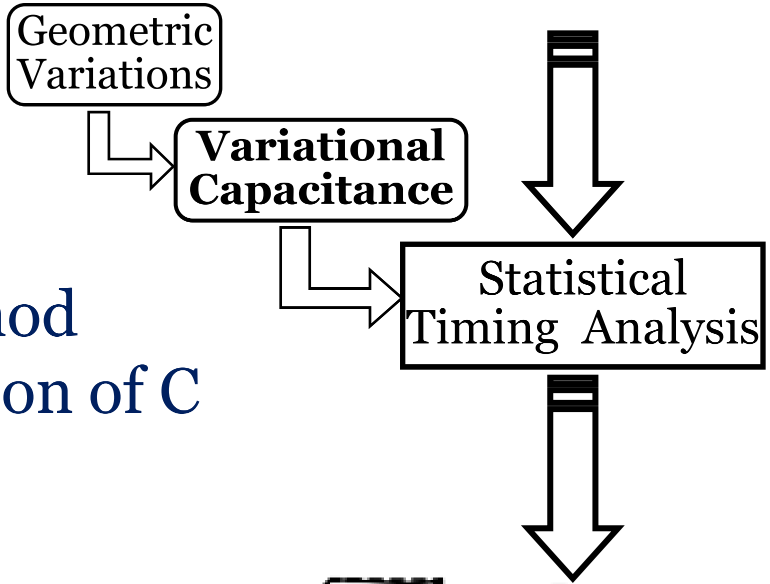

Process Tech.
Layout Pattern structures
2D/3 D Field Solver
Circuit Layout
Pattern Library
Parasitics DB





Background

- Statistical C extraction
    - Systematic variations, random variations
    - Need stochastic modeling method to generate statistical distribution of C



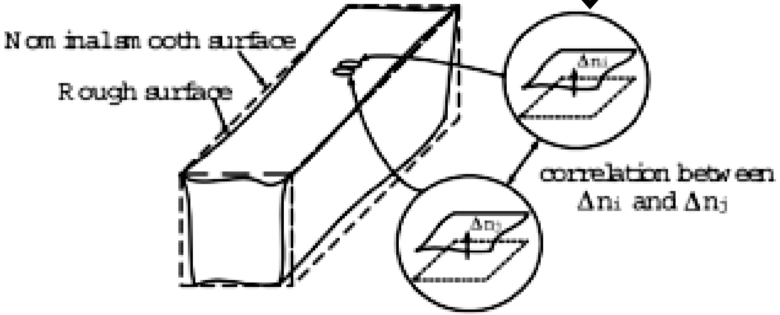
Geometric Variations

Variational Capacitance

Statistical Timing Analysis

- Challenges
    - Accuracy: statistical model, geometric variation model

      geometric parameters obey a spatially correlated multivariate Gaussian distribution



[ICCAD'2005]

-
    - Efficiency: computational expense is thousands of times larger than the non-statistical extraction

Background

- State-of-the-art
  - Monte-Carlo / Quasi-Monte-Carlo: suffers from huge computational time, or not sufficient for the subsequent SCA
  - Perturbation method [ICCAD'05]: quadratic model of C, Taylor's expansion, suitable for small-magnitude variation
  - Spectral stochastic collocation method [DATE'07]: computationally more efficient, a simple geometric model
  - Chip-level HPC method [DATE'08]: considers chip-level extraction problem, with simpler geometric model
  - Continuous-surface method [DAC'09]: continuous-surface geometric model, weighted PFA for acceleration
- A good geometric model should be established to reflect the actual variations, prior to the statistical extraction

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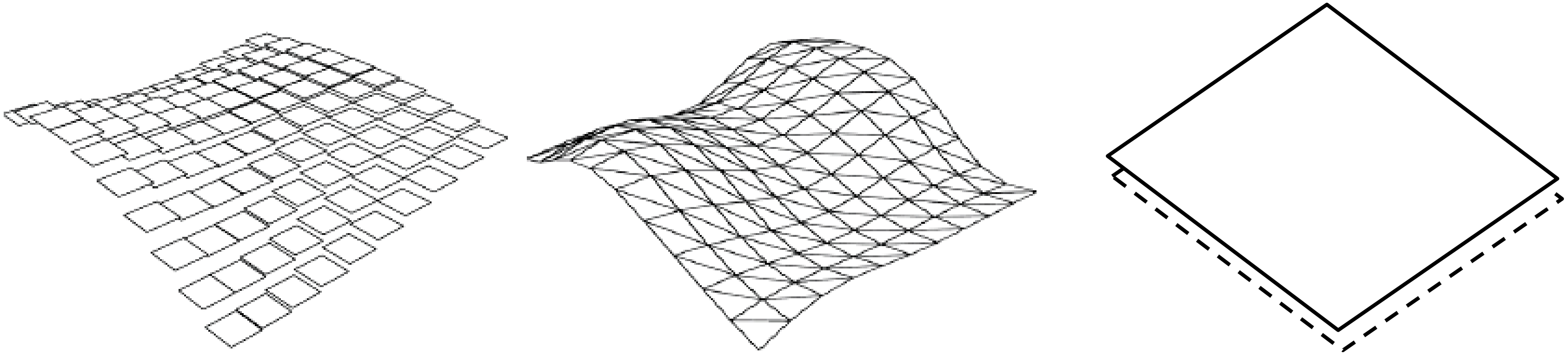
Geometric Variation Models

- Existing geometric variation models
  - **Discontinuous surface variation (DSV):** panels fluctuate along surface’s normal direction, keeping shape unchanged
  - **Continuous surface variation (CSV):** vertices of panels fluctuate differently, and form a continuous surface with triangular panels
  - **Variation as a whole (VAW):** the nominal surface fluctuates as a whole

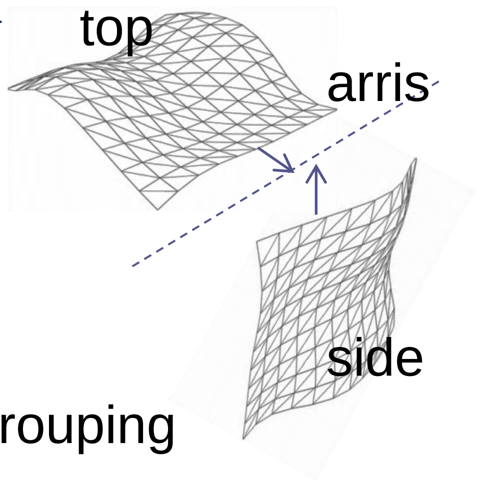


DSV CSV VAW

A variational plane generated with three models

Geometric Variation Models

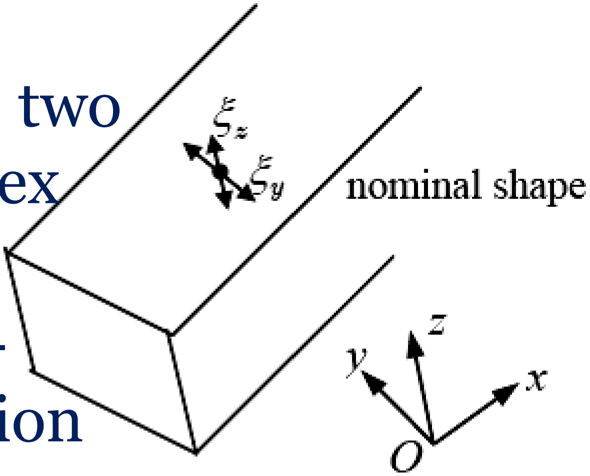
- Existing geometric variation models
  - VAW does not consider the detailed variations, used for sensitivity calculation, or simplified 2-D structure
  - DSV generates incomplete surface, obviously deviates from the actual situation
  - The CSV model proposed in [DAC'09], seems more reasonable than the other two. However, it' s not trivial to depict actual 3-D wire with both width and thickness variations
  - If top and side surfaces fluctuate independently, the shape becomes incomplete or irregular around the arris



We shall consider more on variable setting and grouping

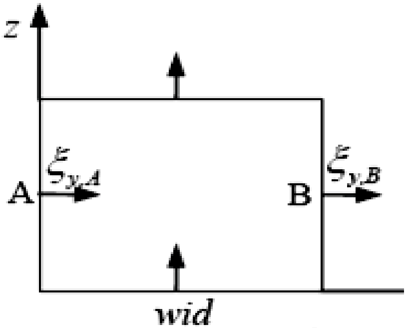
Geometric Variation Models

- The existing CSV model [DAC'09]
  - To avoid the irregularity around arris, two random variables are set for each vertex
  - All the variables are divided into two groups: ξ_y and ξ_z. Each includes correlated variables with same "+" direction
  - The shortages: redundant variables; unreasonably large surface variation for moderate width/thickness variation



$$\xi_W = \xi_{y,B} + wid - \xi_{y,A} \Rightarrow \text{std}(\xi_W) = \sqrt{E(\xi_W^2) - E^2(\xi_W)} = \sqrt{E(\xi_{y,B}^2) + E(\xi_{y,A}^2) - 2\,\text{cov}(\xi_{y,B}, \xi_{y,A})} \approx \sigma_y \cdot \frac{\sqrt{2} \cdot wid}{\eta_y}$$

$\Rightarrow$ $\sigma_y \approx 5.7 \cdot \text{std}(\xi_W)$, if η_y is 8x wid

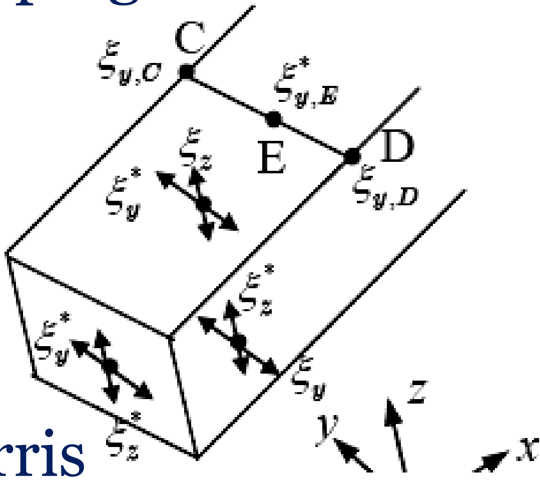


A cross section

$\text{std}(\xi_W) = 10\%$ means $\sigma_y = 57\%$, cause unreasonably large surface fluctuation!

Geometric Variation Models

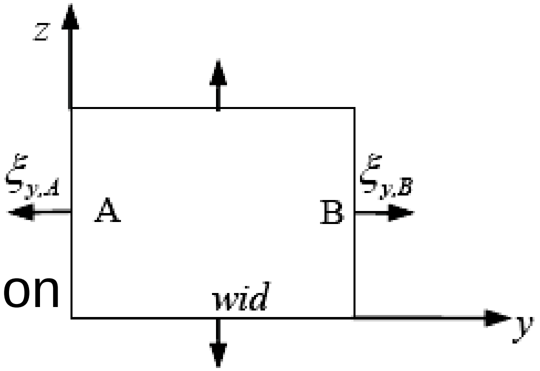
- Improved CSV (ICSV) model
  - New scheme of variable setting and grouping
  - Set only 1 independent random variable for each vertex, along outer normal
  - Tangential displacement of vertex (called derived variable ξ^*) produced by interpolating two variables at edge
  - This guarantees regular shape around arris
  - 4 groups of variables: top, bottom, left-side, and right-side



Calculate the width variation again:

$$\xi_W = \xi_{y,B} + wid + \xi_{y,A} \Rightarrow \mathrm{std}(\xi_W) = \sqrt{2}\sigma_y$$

Can set reasonable σ_y / σ_z to model width/thick variation



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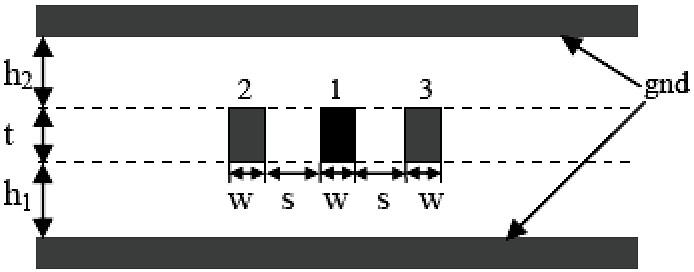
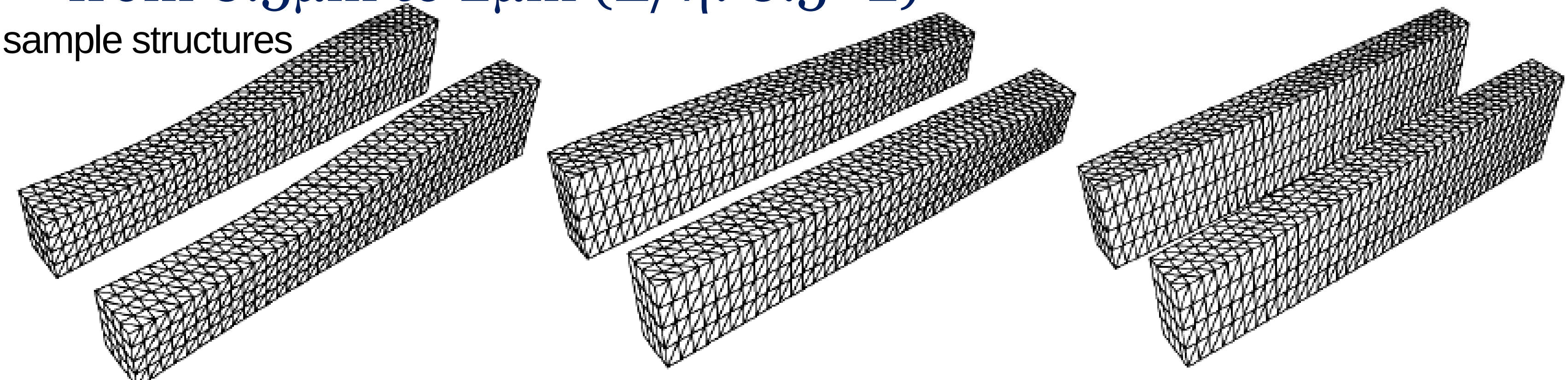
Comparison Experiments

- Experimental settings
  - A typical pattern-library structure
  - 65nm tech., t=h1=h2=0.2μm, w=0.1μm, wire length L=1μm
  - h1 and h2 are two random variables; random surface variation is considered in both width and thickness
  - Variation Std σ=10%; change the correlation length η from 0.5μm to 2μm (L/η: 0.5~2)

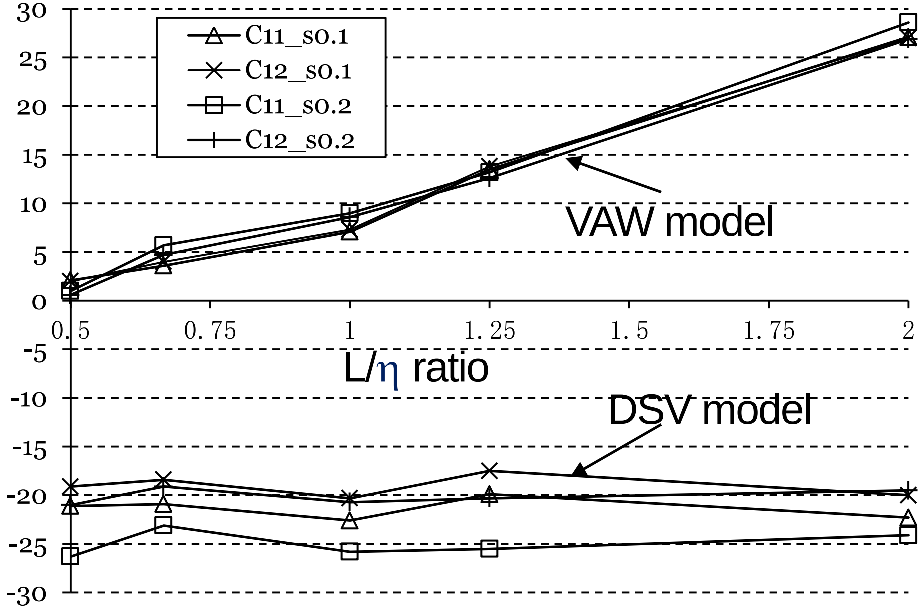


Random sample structures


(a) η =0.5μm
(b) η =1μm
(c) η =2μm

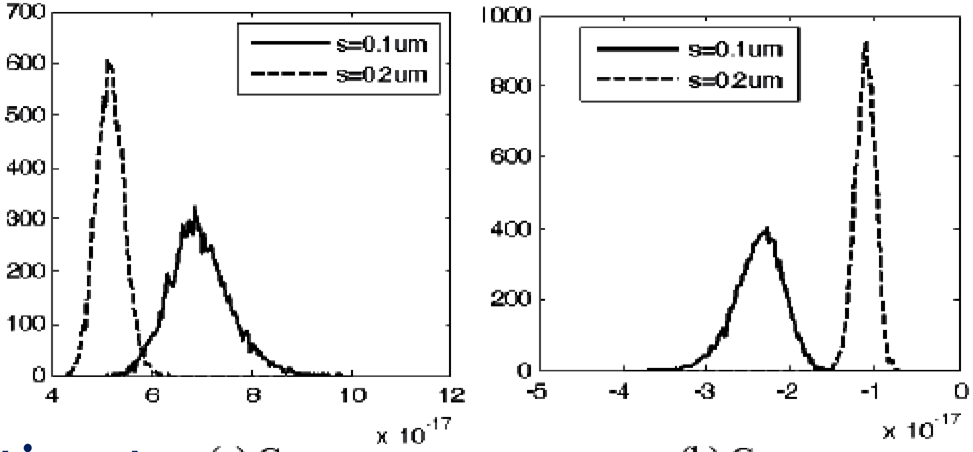
Comparison Experiments

- The errors of VAW and DSV models
  - Depict the geometry with VAW, DSV and ICSV models, and regard the statistical results of MC-10000 in ICSV model as the standard
  - Errors of VAW and DSV in Std of capacitances with L/η changed (s=0.1 and 0.2μm)
  - Error of VAW ∝ L/η, error of DSV is always about -20%



Comparison Experiments

- The statistical distribution of C
    - Std > 5% of mean for C11, and >10% for C12
    - Skewed distribution suggests quadratic model



(a) C_{11} (b) C_{12}

- Three observations
    - OB1: DSV model underestimates the Std of total/coupling capacitances, with ≥20% error.
    - OB2: VAW model overestimates the Std of capacitance, with the error $\propto L/\eta$; it may be valid for small structures.
    - OB3: For structures with Std of the geometric variation to be 10%, the statistical capacitance has skewed distribution requiring quadratic stochastic model.

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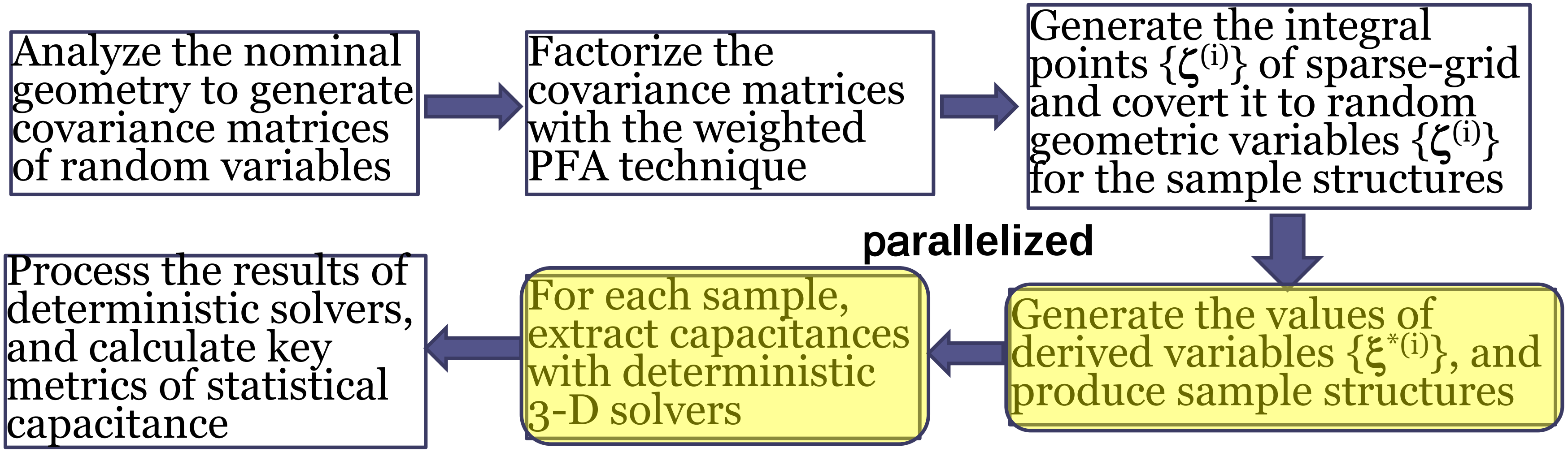
Statistical Capacitance Extraction

- A new statistical capacitance solver - statCap
  - Based on the ICSV model
  - Employ the HPC [DATE'08] for quadratic model

$$\tilde{C}(\zeta) = a_0\Psi^0 + \sum_{i_1=1}^{d} a_{i_1}\Psi^1(\zeta_{i_1}) + \sum_{i_1=1}^{d}\sum_{i_2=1}^{i_1} a_{i_1 i_2}\Psi^2(\zeta_{i_1},\zeta_{i_2})$$

  - Employ the wPFA [DAC'09] for variable reduction
  - Employ parallel computing for acceleration



Analyze the nominal geometry to generate covariance matrices of random variables

Factorize the covariance matrices with the weighted PFA technique

Generate the integral points $\{\zeta^{(i)}\}$ of sparse-grid and covert it to random geometric variables $\{\zeta^{(i)}\}$ for the sample structures

Generate the values of derived variables $\{\xi^{*(i)}\}$, and produce sample structures

parallelized

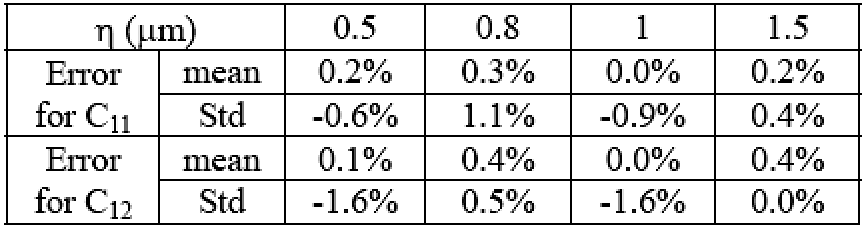
For each sample, extract capacitances with deterministic 3-D solvers

Process the results of deterministic solvers, and calculate key metrics of statistical capacitance

Statistical Capacitance Extraction

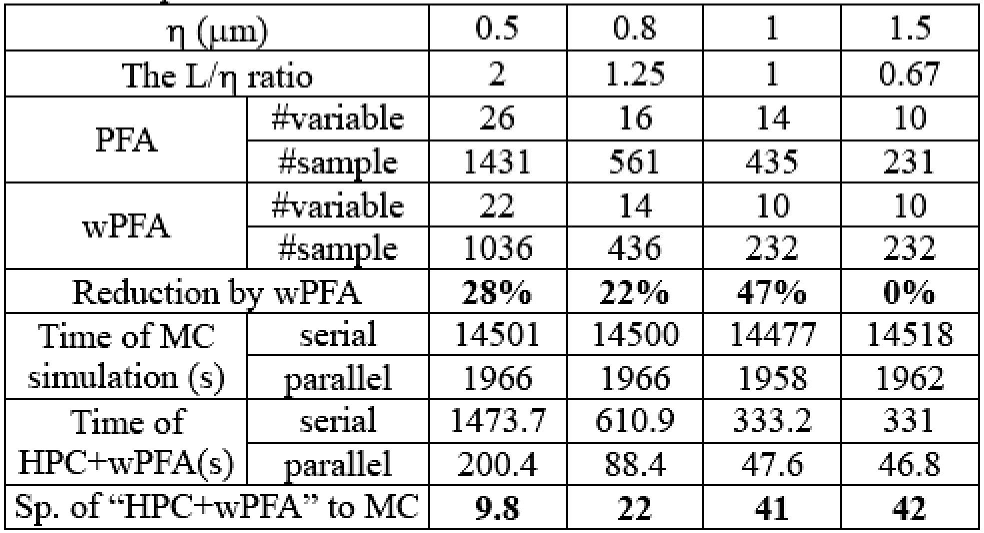
- Numerical results
  - η=0.5, 0.8, 1, 1.5μm; s=0.1μm
  - The results of statCap validate the accuracy of wPFA
  - Efficiency comparisons
  - wPFA reduce 22% time
  - 7x speedup achieved on an 8-core machine
  - Speedup of statCap to MC-10000 varies from 9.8 to 42

The errors of the “HPC+wPFA” method for the four test cases



η (μm)		0.5	0.8	1	1.5
Error for C_{11}	mean	0.2%	0.3%	0.0%	0.2%
	Std	-0.6%	1.1%	-0.9%	0.4%
Error for C_{12}	mean	0.1%	0.4%	0.0%	0.4%
	Std	-1.6%	0.5%	-1.6%	0.0%

The computational results of “HPC+wPFA” and MC simulation



η (μm)		0.5	0.8	1	1.5
The L/η ratio		2	1.25	1	0.67
PFA	#variable	26	16	14	10
	#sample	1431	561	435	231
wPFA	#variable	22	14	10	10
	#sample	1036	436	232	232
Reduction by wPFA		**28%**	**22%**	**47%**	**0%**
Time of MC simulation (s)	serial	14501	14500	14477	14518
	parallel	1966	1966	1958	1962
Time of HPC+wPFA(s)	serial	1473.7	610.9	333.2	331
	parallel	200.4	88.4	47.6	46.8
Sp. of “HPC+wPFA” to MC		**9.8**	**22**	**41**	**42**

Statistical Capacitance Extraction

- Three observations from this experiment
  - OB4: The weighted PFA for statistical capacitance extraction can achieve up to 47% efficiency improvement over the normal PFA, without loss of accuracy.
  - OB5: The HPC-based method can be easily parallelized, and achieves 7X speedup on an 8-core machine.
  - OB6: The HPC-based method is tens of times faster than the MC method for the test structures. For larger structures whose dimension is larger than 2x of correlation length η, its speedup to MC method may be marginal.

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Conclusions

- The contributions of this work
    - An improved continuous surface variation (ICSV) model is proposed to accurately imitate the random geometric variation of on-chip interconnects
    - An efficient parallel statistical capacitance solver
    - With the experiments on a typical 65nm-technology structure, several criteria are drawn regarding the trade-offs of geometric models and statistical methods

Thank you!

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