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Statistical extraction and modeling of inductance considering spatial correlation

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Abstract In this paper, we present a novel method for statistical inductance extraction and modeling for interconnects considering process variations. The new method, called *statHenry*, is based on the collocation-based spectral stochastic method where orthogonal polynomials are used to represent the statistical processes. The coefficients of the partial inductance orthogonal polynomial are computed via the collocation method where a fast multi-dimensional Gaussian quadrature method is applied with sparse grids. To further improve the efficiency of the proposed method, a random variable reduction scheme is used. Given the interconnect wire variation parameters, the resulting method can derive the parameterized closed form of the inductance value. We show that both partial and loop inductance variations can be significant given the width and height variations. This new approach can work with any existing inductance extraction tool to extract the variational partial and loop inductance or impedance. Experimental results show that our method is orders of magnitude faster than the Monte Carlo method for several practical interconnect structures.

Keywords Inductance extraction · Statistical · Spatial correlation · Process variation

This paper is an extended version of the SM2ACD 2010 workshop paper [1].

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1 Introduction

It is well accepted that process-induced variations have a huge impact on circuit performance in sub-100 nm VLSI technologies [2, 3]. A significant portion of these variations are purely random in nature [4]. As a result, variation-aware design methodologies and statistical computer-aided design (CAD) tools are widely believed to be the key to mitigate this grand challenge for 45 nm technologies and beyond [3, 4]. Variational considerations have to be incorporated into every step of the design and verification process to ensure reliable chips and profitable manufacturing yields.

In this paper, we investigate the impact of geometric variations on the extracted inductance values (partial or loop). Parasitic extraction algorithms have been intensively studied in the past to estimate the resistance, capacitance, inductance, and susceptance of 3-D interconnects [5–8]. Many efficient algorithms like the FastCap [7], FastHenry [6] and FastImp [8] were proposed based on the boundary element method (BEM) or volume discretization methods (for partial element equivalent circuit (PEEC) based inductance extraction [5]). In the nanometer regime, circuit layout will have significant variations, both systematic and random, coming from the fabrication process. Some recent research works have been proposed using different variational models for capacitance extraction while considering process variations [9–12]. However, less research has been done on variational inductance extraction in the past.

We propose a new statistical inductance extraction method, called *statHenry*, based on a spectral stochastic collocation scheme. This approach is based on the Hermite orthogonal polynomial representation of the variational inductance. *StatHenry* applies the collocation idea where the inductance extraction process is performed many times in pre-determined sampling positions so that the coefficients of the orthogonal polynomials can be computed using the weighted least square method. The number of samplings is $O(m^2)$, where m is the number of variables for the second order Hermite polynomials. If m is large, the approach will lose its efficiency compared to the Monte Carlo method. To mitigate this problem, a weighted principle factor analysis method is performed to reduce the number of variables by exploiting the spatial correlations of variational parameters. Experimental results show that the new method is orders of magnitudes faster than the Monte Carlo method with very small errors for several practical interconnect structures. We also show that typical variation for the width and height of wires (10–30%) can cause significant variations to both partial and loop inductance.

The rest of this paper is organized as follows: Sect. 2 presents the statistical inductance extraction problem, Sect. 3 reviews the orthogonal polynomial chaos based stochastic sampling methods, and Sect. 4 presents our new statistical inductance extraction method. Then in Sect. 5 we present the experimental results and Sect. 6 concludes this paper.

2 Problem formulation

For a system with m conductors, we first divide all conductors into b filaments. The resistance and inductance of all filaments are respectively stored in matrices $R_{b\times b}$ and $L_{b\times b}$, each with dimensions $b \times b$. R is a diagonal matrix with its diagonal element

$$R_{ii} = \frac{l_i}{\sigma a_i} \tag{1}$$

where l_i is the length of filament i, σ is conductivity and a_i is the area of the cross section of filament i. L is a dense matrix, L_{ij} can be represented as in [6]:

$$L_{ij} = \frac{\mu}{4\pi a_i a_j} \int_{V_i} \int_{V_j} \frac{\mathbf{l_i l_j}}{\|\mathbf{r} - \mathbf{r}'\|} dV_i dV_j \tag{2}$$

where μ is permeability, $\mathbf{l}_i$ and $\mathbf{l}_j$ are unit vectors of the lengthwise direction of filaments i and j, $\mathbf{r}$ is an arbitrary point in the filament, and V_i and V_j are the volumes of filaments i and j, respectively. Assuming magnetoquasistatic electric fields, the inductance extraction problem is then finding the solution to the discretized integral equation:

$$\begin{aligned}\left(\frac{l_i}{\sigma}\right) I_i + j\omega \sum_{j=1}^{b} \left(\frac{\mu}{4\pi a_i a_j} \int_{V_i} \int_{V_j} \frac{\mathbf{l_i l_j}}{\|\mathbf{r} - \mathbf{r}'\|} dV_i dV_j \right) I_j \\ = \frac{1}{a_i} \int_{a_i} (\Phi_A - \Phi_B) dA\end{aligned} \tag{3}$$

where I_i and I_j are the currents inside the filaments i and j, ω is the angular frequency, and Φ_A and Φ_B are the potentials at the end faces of the filament. Equation 3 can be written in the matrix format as

$$(R + j\omega L) I_b = V_b \tag{4}$$

where $I_b \in \mathcal{C}^b$ is the vector of b filament currents, V_b is a vector of dimension b containing the filament voltages. We will first solve for the inductance between one conductor, which we will call the primary conductor, and all others, which we will call the environmental conductors. To do this, we set the voltages of filaments in our primary conductor to unit voltage and voltages of all other filaments to zero. Therefore I_b can be calculated by solving a system of linear equations. Together with the current conservation (Kirchhoff's current law) equation

$$M I_b = I_m \tag{5}$$

on all the filaments, where M is an adjacent matrix for the filaments and I_m is the currents of all m conductors. By repeating this process with each of the m conductors as the primary conductor, we can obtain $I_{m,i}, i = [1, \ldots m]$ vectors which form a $m \times m$ matrix $I_p = [I_{m,1}, I_{m,2}, \ldots, I_{m,m}]$. Since the voltages of all primary conductors have been set to unit voltage previously, the resistance and inductance can be achieved respectively from the real part and the imaginary part of the inverse matrix of I_p. The authors in [6] proposed a hierarchical multipole algorithm FastHenry to solve (4) (5). The FastHenry is efficient as it is proved to be one order of magnitude faster than solving (4) (5) using tradition methods and retain the same accuracy. Since our proposed *statHenry* is sampling based, we run the FastHenry package [13] to obtain the extracted inductance value on each sample.

Process variations affecting conductor geometry are reflected by changes in the width and height of the conductors. We ignore the length of the wires as the variations are typically insignificant compared to its magnitude. These variations will make each element in the inductance matrix follow some kinds of random distributions. Solving this problem is done by deriving the random distribution and then effectively computing the mean and variance of the inductances with the given geometric randomness parameters. In this paper, we assume that width and height in each filament i are disturbed by random variables $\Delta n_{w,i}$ and $\Delta n_{h,i}$, which gives us:

$$w_i{}' = w_i + \Delta n_{w,i} \quad (6)$$

$$h_i{}' = h_i + \Delta n_{h,i} \quad (7)$$

where the size of Δx_i is a Gaussian distribution $|\Delta x_i| \sim N(0, \sigma^2)$. The correlation between random perturbations on each wire's width and height are governed by an empirical formulation such as the widely used exponential model

$$\gamma(r) = e^{-r^2/\eta^2} \quad (8)$$

where r is the distance between two panel centers and η is the correlation length. The most straightforward method is to use a Monte Carlo(MC) based simulation to obtain distribution, mean, and variance of all those inductances. Unfortunately, the MC method will be extremely time consuming and more efficient statistical approaches are needed.

3 Review of spectral stochastic based methods

In this section, we briefly review the spectral stochastic and orthogonal polynomial chaos (PC) based stochastic analysis methods.

In the following, $\xi(\theta)$ is a random variable expressed as a function of θ, which is the random event. Hermite PC (HPC) utilizes a series of polynomials, which are orthogonal with respect to the Gaussian distribution, to facilitate stochastic analysis [14, 15]. These polynomials are used as an orthogonal basis to decompose a random process.

We note that for Gaussian and log-normal distributions, using Hermite polynomials are the best choice, as they lead to an exponential convergence rate [14]. For distributions which are neither Gaussian nor log-normal, there are other orthogonal polynomials such as Legendre for uniform distribution, Charlier for Poisson distribution, and Krawtchouk for Binomial distribution, etc [16, 17].

To simplify the explanation, only one random variable is considered and the one-dimensional Hermite polynomials are expressed as follows:

$$H_0^1(\xi) = 1, H_1^1(\xi) = \xi, H_2^1(\xi) = \xi^2 - 1, H_3^1(\xi) = \xi^3 - 3\xi, \ldots \quad (9)$$

The Hermite polynomials are orthogonal with respect to a Gaussian weighted expectation (the superscript n is dropped to simplify notation):

$$\langle H_i(\xi), H_j(\xi) \rangle = \langle H_i^2(\xi) \rangle \delta_{ij}, \quad (10)$$

where δ_{ij} is the Kronecker delta and $\langle *, * \rangle$ denotes an inner product, defined as:

$$\langle f(\xi), g(\xi) \rangle = \frac{1}{\sqrt{(2\pi)^n}} \int f(\xi) g(\xi) e^{-\frac{1}{2}\xi^T \xi} d\xi. \quad (11)$$

Given a random variable $v(\xi)$, where $\xi = [\xi_1, \ldots, \xi_n]$ denotes a vector of orthonormal Gaussian random variables with zero mean, the random variable can be approximated using a truncated Hermite PC expansion [14]:

$$v(\xi) \approx \sum_{k=0}^{P} a_k H_k^n(\xi), \quad (12)$$

where n is the number of independent random variables, $H_k^n(\xi)$ are n-dimensional Hermite polynomials, and a_k are the deterministic coefficients. The number of terms, P, is given by

$$P = \sum_{k=0}^{p} \frac{(n-1+k)!}{k!(n-1)!}, \quad (13)$$

where p is the order of the Hermite PC. According to the Galerkin method, the truncation error is minimized [14] when

$$\langle v(\xi), H_k(\xi) \rangle = \langle \sum_{j=1}^{P} a_j H_j(\xi), H_k(\xi) \rangle. \quad (14)$$

Based on the orthogonality of Hermite polynomials, the right part of the above equation equals zero when $j \neq k$. The coefficients, a_k, can then be represented as:

$$a_k(t) = \frac{\langle v(\xi), H_k(\xi) >}{< H_k^2(\xi) \rangle}, \forall k \in \{0, \ldots, P\}. \quad (15)$$

The key issue is to compute the coefficients $a_k(t)$ with an efficient numerical integration method, which is discussed in the next section.

4 New statistical inductance extraction method—*statHenry*

In this section, we present the new statistical inductance extraction method—*statHenry*. The new method is based on stochastic analysis method where the integration of (11) in (15) is computed via an improved numerical quadrature method. Our new method is based on the efficient multi-dimensional numerical Gaussian quadrature. We will first review the numerical Gaussian quadrature method, followed by the improved Smolyak quadrature.

4.1 Gaussian quadrature technique

The Gaussian quadrature method is an efficient numerical method for computing the definite integral of a function

[18]. Using this method, we can compute the coefficients $a_k(t)$ in (15). Next, we will review this method, which uses the Hermite polynomial shown below.

Our goal is to determine the numerical solution to the integral equation $\langle x(\xi), H_j(\xi)\rangle$. In our problem, this is a one-dimensional numerical quadrature problem based on Hermite polynomials [18]. Thus, we have

$$\begin{aligned}\langle x(\xi), H_k(\xi)\rangle &= \frac{1}{\sqrt{(2\pi)}}\int x(\xi)H_k(\xi)e^{-\frac{1}{2}\xi^2}d\xi \\ &\approx \sum_{i=0}^{P} x(\xi_i)H_i(\xi_i)w_i\end{aligned} \tag{16}$$

Here we have only a single random variable ξ. ξ_i and w_i are Gaussian Hermite quadrature abscissas (quadrature points) and weights.

The Quadrature rule states that if we select the roots of the Pth Hermite Polynomial as the quadrature points, the quadrature is exact for all polynomials of degree $2P - 1$ or less for (16). This is called $(P - 1)$-level accuracy of the Gaussian-Hermite quadrature.

For multiple random variables, a multi-dimensional quadrature is required. The traditional way of computing a multi-dimensional quadrature is to use a direct tensor product based on one dimensional Gaussian Hermite quadrature abscissas and weights [19]. With this method, the number of quadrature points needed for n-dimensions at level P is about $(P + 1)^n$, which is well known as the curse-of-dimensionality.

4.2 Sparse grid technique

Smolyak quadrature [19], also known as sparse grid quadrature, is used as an efficient method to reduce the number of quadrature points. Let us define a one-dimensional sparse grid quadrature point set $\Theta_1^P = \{\gamma_1, \gamma_2, \ldots, \gamma_P\}$, which uses $P + 1$ points to achieve degree $2P + 1$ of exactness. The sparse grid for an n-dimensional quadrature at degree P chooses points from the following set:

$$\Theta_n^P = \bigcup_{P+1 \le |\mathbf{i}| \le P+n} \left(\Theta_1^{i_1} \times \ldots \times \Theta_1^{i_n}\right) \tag{17}$$

where $|\mathbf{i}| = \sum_{j=1}^{n} i_j$. The corresponding weight is:

$$w_{j_{i_1} \cdots j_{i_n}}^{i_1 \ldots i_n} = (-1)^{P+n-|\mathbf{i}|}\binom{n-1}{n+P-|\mathbf{i}|}\prod_m w_{j_{i_m}}^{i_m} \tag{18}$$

where $\binom{n-1}{n+P-|\mathbf{i}|}$ is the combinatorial number and w is the weight for the corresponding quadrature points. It has been shown that interpolation on a Smolyak grid ensures a bound for the mean-square error [19]

$$|E_P| = O\left(N_P^r(logN_P)^{(r+1)(n-1)}\right),$$

where N_P is the number of quadrature points and k is the order of the maximum derivative that exist for the delay function. The number of quadrature points increases as $O\left(\frac{n^P}{(P)!}\right)$.

It can be shown that a sparse grid of at least level P is required for an order P representation. The reason is that the approximation contains order P polynomials for both $x(\xi)$ and $H_j(\xi)$. Thus, there exists $x(\xi)H_j(\xi)$ with order $2P$, which requires a sparse grid of at least level P with an exactness degree of $2P + 1$.

Therefore, level 1 and level 2 sparse grids are required for linear and quadratic models, respectively. The number of quadrature points is about $2n$ for the linear model, and $2n^2$ for the quadratic model. The time cost is about the same as the Taylor-conversion method, while keeping the accuracy of homogenous chaos expansion.

In addition to the sparse grid technique, we also employ several accelerating techniques. Firstly, when n is too small, the number of quadrature points for sparse grid may be larger than that of direct tensor product of a Gaussian quadrature. For example, if there are only 2 variables, the number is 5 and 15 for level 1 and 2 sparse grid, compared to 4 and 9 for direct tensor product. In this case, the sparse grid will not be used. Secondly, The set of quadrature points (17) may contain the same points with different weights. For example, the level 2 sparse grid for 3 variables contain 4 instances of the point (0,0,0). Combining these points by summing the weights reduces the computational cost of $x(\gamma_i)$.

4.3 Variable decoupling and reduction

Even with sparse grid quadrature, the number of sampling points still grow quadratically with the number of variables. As a result, we should further reduce the number of variables by exploiting the spatial correlations of the given random width and height parameters of wires.

We start with independent random variables as the input of the spectral stochastic method. Since the height and width variable of all wires are correlated, this correlation should be removed before using the spectral stochastic method. We first present the following result as our theoretical basis for decoupling the correlation of those variables [20].

Proposition 1 *For a set of zero-mean Gaussian distributed variables ξ^* whose covariance matrix is Δn, if there is a matrix L satisfying $\Delta n = LL^T$, then ξ^* can be represented by a set of independent standard normal distributed variables ξ as $\xi^* = L\xi$.*

Note that the solution for decoupling is not unique. For example, Cholesky decomposition can be used to seek L since the covariance matrix Δn is always a semi-positive definite matrix. However Cholesky decomposition cannot reduce the number of variables. Principle factor analysis (PFA) [10] can substitute Cholesky decomposition when variable reduction is needed. Eigen-decomposition on the covariance matrix yields:

$$\Delta n = LL^T, L = \left(\sqrt{\lambda_1}e_1, \ldots, \sqrt{\lambda_n}e_n\right), \tag{19}$$

where $\{\lambda_i\}$ are eigenvalues in order of descending magnitude, and $\{e_i\}$ are corresponding eigenvectors. PFA reduces the number of components in ξ by truncating L using the first k items.

The error of PFA can be controlled by k:

$$err = \frac{\sum_{i=k+1}^{n} \lambda_i}{\sum_{i=1}^{n} \lambda_i}, \tag{20}$$

where bigger k leads to a more accurate result. PFA is efficient, especially when the correlation length is large. In our experiments, we set the correlation length being 8 times of width of wires. As a result, PFA can reduce the number of variables from 40 to 14 with an error of about 1% in an example with 20 parallel wires.

4.4 Variable reduction by weighted PFA

Principle factor analysis for variable reduction considers only the spatial correlation between wires, while ignoring the influence of the inductance itself. One idea is to consider the importance of the outputs during the reduction process. We follow the recently proposed weighted PFA (wPFA) technique to seek better variable reduction efficiency [21].

If a weight is defined for each physical variable ξ_i, to reflect its impact on the output, then a set of new variables ξ^* are formed:

$$\xi^* = W\xi \tag{21}$$

where $W = diag(w_1, w_2, \ldots, w_n)$ is a diagonal matrix of weights. As a result, the covariance matrix of $\xi^*, \Delta n(\xi^*)$, now contains the weight information and performing PFA on $\Delta n(\xi^*)$ leads to the weighted variable reduction. Specifically, we have

$$\Delta n(\xi^*) = E(W\xi(W\xi)^T) = W\Delta n(\xi)W^T \tag{22}$$

and denote its eigenvalues and eigenvectors by λ_i^* and e_i^*. Then, the variables ξ can be approximated by the linear combination of a set of independent dominant variables ζ^* :

$$\xi = W^{-1}\xi^* \approx W^{-1}\sum_{i=1}^{k}\sqrt{\lambda_i^*}e_i^*\zeta_i^*. \tag{23}$$

The error controlling process is similar to (20), but using the weighted eigenvalues λ_i^*. For inductance extraction, we take the partial inductance of the deterministic structure as the weight, since this normal structure reflects an approximate equality of inductance compared with the variational structure. By performing wPFA in the same example with 20 parallel wires, 40 variables can now be reduced to 8 rather than 14 when using PFA (more details in the experimental results).

4.5 New extraction algorithm: *statHenry*

After introducing all the important pieces from related works, we are now ready to present our new algorithm—*statHenry*. Figure 1 is a flowchart of the proposed algorithm.

5 Experimental results

In this section, we compare the results of the proposed *statHenry* method against the Monte Carlo method and a simple orthogonal polynomial based spectral stochastic collocation method with the sparse grid technique but without variable reduction, called *HPC*. The proposed method *statHenry* has been implemented in Matlab 8.0. All the experimental results were obtained using a computer with a 1.6 GHz Intel Quad Core i7-720 and 4 GB memory running Microsoft Windows 7 Ultimate operating system. We use FastHenry [6, 13] to do inductance extraction for each sample and the version of FastHenry is 3.0.

In our experiment, we set up four test casts to examine our algorithm: 2 parallel wires, 5 parallel wires, 10 parallel wires and 20 parallel wires (shown in Fig. 2). In all four models, all of the wires have a width of 1 µm, length of

Algorithm: STATHENRY
Input: Wires with variational width and heights
Output: The Hermite polynomial coefficients of the partial or loop inductance values of the wires, $L(\xi)$

1. Perform variable reduction based on weighted PFA.
2. Generate the n-dimensional Smolyak quadrature point sets of second order Θ_n^2 and corresponding weight set w_n.
3. For i = 1 to size(Θ_n^2)
4.   Perform FastHenry for each sample.
5. end
6. Compute the coefficients of Hermite polynomials for the partial or loop inductance values $d(\xi)$

Fig. 1 The proposed *statHenry* algorithm

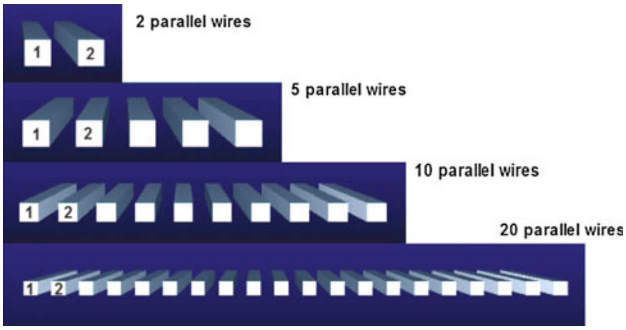

Fig. 2 Four test structures used for comparison

6 µm, and pitch between them of 1 µm. The unit of the inductances in the experiment results is pico-henry (pH).

We set the standard deviation as 10% of the wire widths and wire heights and the correlation length η being 8 µm to indicate a strong correlation.

First, we compare the accuracy of the three methods in terms of the mean and standard deviations of loop/partial inductances. The results are summarized in Table 1. In the table we report the results from four test cases as mentioned. In each case, we report the results for partial self inductance on wire 1 ($L11_p$), and loop inductance between wire 1 and 2 ($L12_l$). Columns 3–4 are the mean value, standard deviation value for the Monte-Carlo method (*MC*). And columns 5–12 are the mean value, standard deviation value and their errors comparing with Monte Carlo method for the simple orthogonal polynomial based method (*HPC*) and the new method. The average error of the mean and standard deviation of HPC method is 0.05 and 2.01% compared with Monte Carlo method while that of *statHenry* method is 0.05 and 2.06%, respectively. The Monte Carlo results comes from 10,000 FastHenry runs.

It can be seen that *statHenry* is very accurate for both mean and standard deviation compared with the *HPC* method and Monte Carlo method. We observe that a 10% standard deviation for the width and height results in variations from 2.73 to 5.10% for the partial and loop inductances, which is significant for timing.

Next, we show the CPU time speedup of the proposed method. The results are summarized in Table 2. It can be seen that *statHenry* can be about two orders of magnitude faster than the Monte Carlo method. The average speedup of the HPC method and *statHenry* method are 54.1 and 349.7 compared with Monte Carlo method. We notice that with more wires, the speedup goes down. It is expected as more wires leads to more variables, even after the variable reduction. And the number of samplings in the collocation method are $O(m^2)$ for second-order Hermit polynomials, where m is the number of variables. As a result, the speedup goes down as more samplings are needed to compute the coefficients while Monte-Carlo has the fixed number of samplings (10,000 for all cases).

Table 3 shows the reduction effects using PFA and wPFA for all the cases under the same errors. We can see that with weighted PFA (wPFA), we can achieve lower reduced variable number and fewer quadrature points for sampling thus better efficiency for the entire extraction algorithm.

Finally, we study the variational impacts of partial and loop inductances under different variabilities for width and height using *statHenry* and the MC method.

The variation statistics are summarized in Table 4. Here we report the results for standard deviations from 10 to 30% for width and height for *statHenry* method and Monte Carlo method for 10 parallel wire case. As the variation due to process imperfections grow as the technology advances, we can see that inductance variation will also grow. Considering a typical 3σ range for variation, a 30% standard deviation means that width and height changes can reach 90% of their values. It can be seen that with the increasing variations of width and height (from 10 to 30%), the *std/mean* of partial inductance grows from 2.75 to 8.65% while that of loop inductance grows from 5.10 to 15.9%, which can significantly impact the noise and delay of the wires. The average error of mean and standard deviation of *statHenry* is 0.33 and 1.75% compared with Monte Carlo for all variabilities of width and height. From

Table 1 Accuracy comparison (mean and variance values of inductances) among Monte Carlo, HPC and *statHenry*

Wires	Inductance	Monte Carlo		HPC		Error		*statHenry*		Error	
		Mean (pH)	Std (pH)	Mean (pH)	Std (pH)	Mean (%)	Std (%)	Mean (pH)	Std (pH)	Mean (%)	Std (%)
2	$L11_p$	2.851	0.080	2.850	0.078	0.02	2.31	2.850	0.078	0.03	2.47
2	$L12_l$	3.058	0.158	3.057	0.156	0.05	1.50	3.056	0.155	0.06	2.21
5	$L11_p$	2.849	0.078	2.851	0.078	0.08	0.86	2.851	0.078	0.07	0.24
5	$L12_l$	3.054	0.155	3.058	0.156	0.11	1.01	3.058	0.156	0.11	0.70
10	$L11_p$	2.852	0.079	2.853	0.078	0.01	1.23	2.853	0.078	0.02	1.37
10	$L12_l$	3.059	0.159	3.060	0.156	0.05	1.55	3.060	0.156	0.05	1.74
20	$L11_p$	2.852	0.081	2.853	0.078	0.03	3.74	2.853	0.078	0.03	3.82
20	$L12_l$	3.059	0.163	3.060	0.156	0.04	3.88	3.060	0.156	0.05	3.96

Table 2 CPU Runtime comparison among MC, HPC and *statHenry*

Wires	MC Time (s)	HPC Time (s)	Speedup (vs MC)	*statHenry* Time (s)	Speedup (vs MC)
2	5394.4	32.6	165.4	9.8	550.4
5	7442.8	192.5	38.7	12.6	589.1
10	8333.5	893.7	9.3	42.5	195.9
20	13698.3	4532.9	3.0	215.8	63.5

Table 3 Reduction effects of PFA and wPFA

Wires	Original	PFA		wPFA	
	Variables	Reduction	Points	Reduction	Points
2	4	4	45	2	15
5	10	4	45	2	15
10	20	6	91	4	45
20	40	14	435	8	153

Table 4 Variation impacts on inductances using *statHenry*

variation	Monte Carlo***		*statHenry*		Error	
	Mean	Std	Mean	Std	Mean (%)	Std (%)
10 parallel wires $L11_p$ (pH)						
10%	2.852	0.079	2.853	0.078	0.02	1.37
20%	2.872	0.163	2.862	0.160	0.35	1.84
30%	2.890	0.245	2.879	0.249	0.36	1.45
10 parallel wires $L12_l$ (pH)						
10%	3.059	0.159	3.060	0.156	0.05	1.74
20%	3.097	0.325	3.078	0.319	0.61	1.84
30%	3.128	0.484	3.110	0.495	0.56	2.26

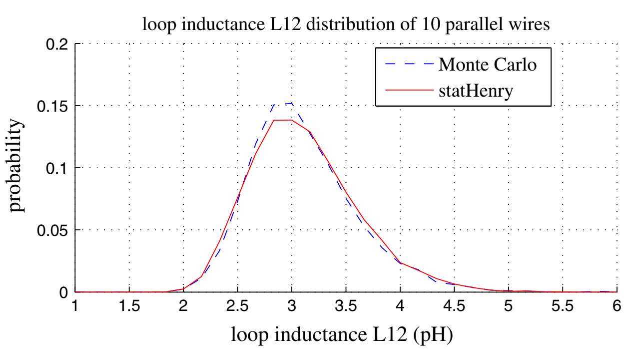

Fig. 3 The loop inductance $L12_l$ distribution changes for the 10 parallel wire case under 30% width and height variations

this, we can see that the results of *statHenry* agree closely with MC under different variations.

Figures 3 and 4 show the loop (for wire 1 and wire 2, $L12_l$) and partial inductance distributions (for wire 1 itself, $L11_p$) under 30% deviations of width and heights for the 10 parallel wire case.

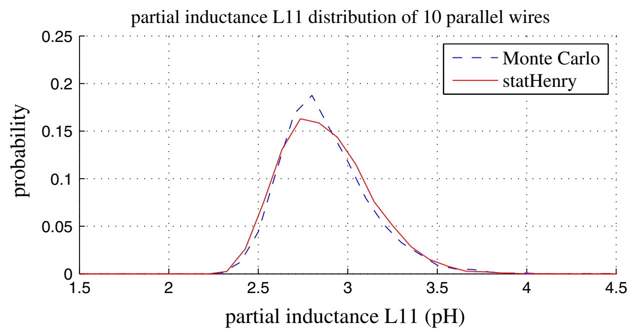

Fig. 4 The partial inductance $L11_p$ distribution changes for the 10 parallel wire case under 30% width and height variations

6 Conclusion

In this paper, we have proposed a new statistical inductance extraction method, called *statHenry*, for interconnects considering process variations with spatial correlation. This new method is based on the collocation-based spectral stochastic method where orthogonal polynomials are used to represent the variational geometrical parameters in a deterministic way. Statistical inductance values are then computed using a fast multi-dimensional Gaussian quadrature method with sparse grid technique. Then, to further improve the efficiency of the proposed method, a random variable reduction scheme based on weighted principle factor analysis is applied. Experimental results show that our method is orders of magnitudes faster than the Monte Carlo method with very small errors for several practical interconnect structures. We also show that both partial and loop inductance variations can be significant for the typical 10–30% standard variations of width and heights of interconnect wires.

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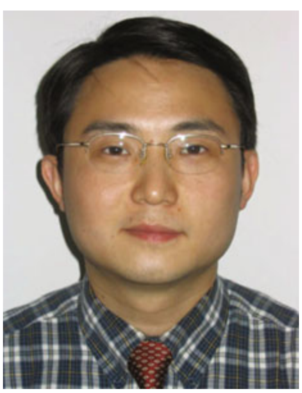

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